List of Courses

Department of Mathematics and Statistics
Indian Institute of Science Education and Research Kolkata
To be implemented from 18MS batch of students.

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The Aim of This Syllabus

This document describes the courses offered by the Department of Mathematics and Statistics (DMS) at Indian Institute of Science Education and Research (IISER) Kolkata. The idea is to provide a holistic introduction to mathematics with all the basic areas thoroughly and rigorously emphasized in the training. It is hoped that an informed choice from a pool of subjects with strong foundations would propel students in their future paths. The syllabi have been modified from second year onwards. This change is implemented for the second year students in Autumn Semester 2019.

A few salient features of the curriculum are the following:

- *Core Courses:* These consist of Analysis (Real Analysis, Complex Analysis, Functional Analysis), Algebra, Geometry, Graph Theory, Linear Algebra. It is desirable that a student in Mathematics masters these topics well.
- *Electives:* The Department offers a diverse bouquet of courses that range from introductory topics to the frontiers of current research. Note that the electives are structured into two thematic directions – one is directed towards mathematics while the other is towards statistics. In each semester, from 3rd year onwards, one may take electives from other departments as well in order to get a minor in other discipline.
- The prerequisite(s) has (have) been mentioned against each of the courses. However, the UGAC convener, in consultation with the instructor, may allow any student without prerequisite if the instructor thinks that the student has enough knowledge to take the course.
- All 4-credit theory courses will have 3 hours of theory and 1 hour of tutorial.

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DMS Course Structure for BS-MS Dual Degree Program
IISER Kolkata

1st Semester:

- MA1101: Mathematics I (3 credits)

2nd Semester:

- MA1201: Mathematics II (3 credits)
- MA1202: Mathematical Methods I (3 credits)

3rd Semester:

- MA2101: Analysis I (4 credits)
- MA2102: Linear Algebra I (3 credits)
- MA2103: Mathematical Methods II (3 credits)

4th Semester:

- MA2201: Analysis II (4 credits)
- MA2202: Probability I (4 credits)

5th Semester: 20 credits

- MA3101: Analysis III (4 credits)
- MA3102: Algebra I (4 credits)
- MA3103: Introduction to Graph Theory & Combinatorics (4 credits)
- MA3104: Linear Algebra II (4 credits)
- MA3105 (Elective): Numerical Analysis (4 credits)/
  Elective from other departments (4 credits)

6th Semester: 20 Credits

- MA3201: Topology (4 Credits)
- MA3202: Geometry of Curves and Surfaces/ MA3205: Statistics I/
  Elective from other Departments(4 Credits). To choose any two.
- MA3203: Algebra II (4 Credits)
- MA3204: Analysis IV (4 Credits)

7th Semester: 20 credits

Stream M

- MA4101: Algebra III (4 credits)
- MA4102: Functional Analysis (4 credits)
- MA4103: Analysis V (4 credits)
- MA4104: Algebraic Topology (4 credits)
- MA4105 (Elective): Elementary Number Theory/MA4106 (Elective): Statistics II/ Elective from other departments (4 credits)

Stream S

- MA4101: Algebra III (4 credits)
- MA4102: Functional Analysis (4 credits)
- MA4103: Analysis V (4 credits)
- MA4107: Statistical Inference (4 credits)
- MA4105 (Elective): Elementary Number Theory/MA4106 (Elective): Statistics II/ Elective from other departments (4 credits)

8th Semester: 20 credits

Stream M

- MA4201: Complex Analysis (4 credits)
- MA4202: Ordinary Differential Equations (4 credits)
- MA4203: Probability II (4 credits)
- MA4205: Differential Geometry (4 credits)
- MA4204 (Elective): Representation Theory of Finite Groups (4 credits)/ MA4207 (Elective): Machine Learning & Network Analysis (4 credits)/ MA4208 (Elective): Independent Study I/ Elective from other departments

Stream S

- MA4201: Complex Analysis (4 credits)
- MA4202: Ordinary Differential Equations (4 credits)
- MA4203: Probability II (4 credits)
- MA4206: Linear Models (4 credits)
- MA4204 (Elective): Representation Theory of Finite Groups (4 credits)/ MA4207 (Elective): Machine Learning & Network Analysis (4 credits)/ MA4208 (Elective): Independent Study I/ Elective from other departments

9th Semester: 20 credits

- MA5101: MS Project I (8 credits)
- MA5102: Partial Differential Equations (4 credits)
- Elective I (4 credits)
- Elective II (4 credits)/ Elective from other departments

Electives I and II have to be chosen from Group A from the list of electives given in the following page.

10th Semester: 20 credits

- MA5201: MS Project II (12 credits)
- Elective III (4 credits)
- Elective IV (4 credits)/Elective from other departments

Electives III and IV have to be chosen from Group B from the list of electives given in the followong page.

Group of Electives

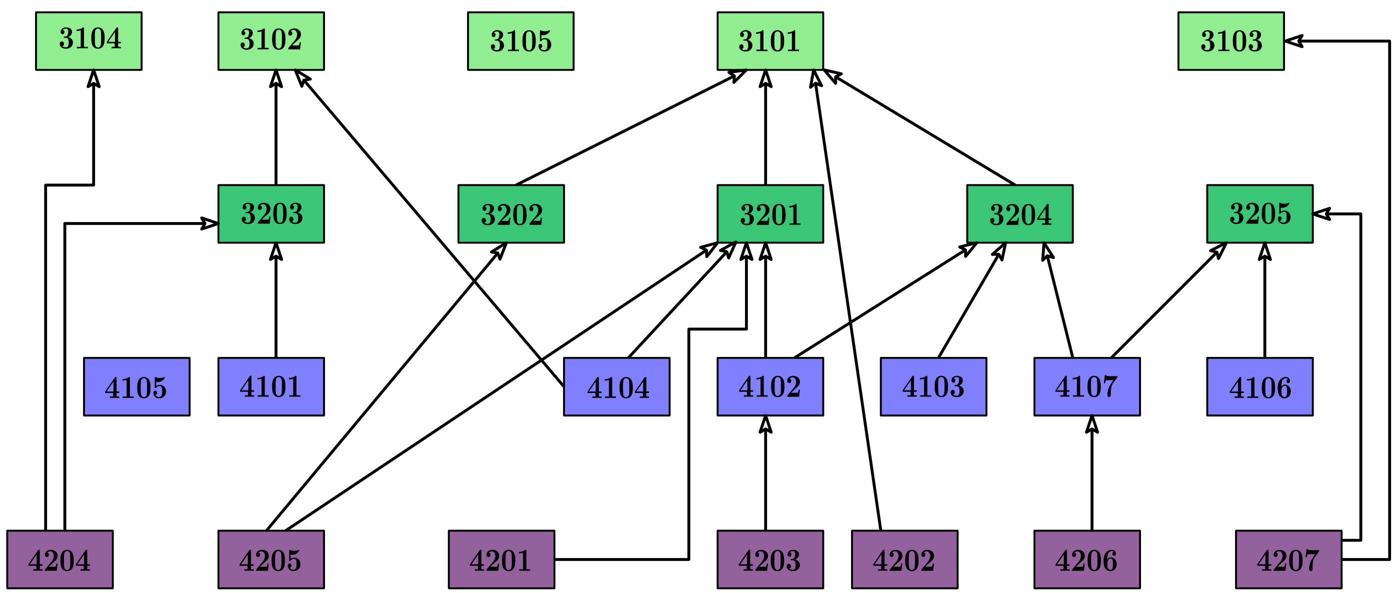
Group A	Fourier Analysis (MA5103) Operator Theory (MA5104) Algebraic Number Theory (MA5105) Topics in Complex Analysis (MA5106) Stochastic Processes (MA5107) Multivariate Statistics (MA5108) Nonparametric Statistics (MA5109) Introduction to Lie Algebra (MA5110) Statistical Decision Theory (MA5111) Theory of Sample Surveys (MA5112) Reliability Theory (MA5113) Riemannian Geometry (MA5114) Independent Study II (MA5115) Introduction to PDE and Distribution Theory (MA5116) Introduction to Ergodic Theory (MA5117) Commutative Algebra (MA5118)
Group B	Algebraic Geometry (MA5202) Topics in Operator Theory (MA5203) Several Complex Variables (MA5204) Advanced Partial Differential Equations (MA5205) Topics in Analysis (MA5206) Topology and Geometry (MA5207) Introduction to Bayesian Analysis (MA5208) Time Series Analysis (MA5209) Advanced Data Structures and Algorithms (MA5210) High Dimensional Statistics (MA5211) Regression Analysis (MA5212) Sobolev Spaces: Theory and Applications (MA5213) Principal Bundles & Representation Ring (MA5214) Analytic Number Theory (MA5215) Independent Study III (MA5216) Independent Study IV (MA5217)

DMS Course Dependency Diagram

The diagram indicates which lower level courses are prerequisite(s) for a particular DMS course in 3rd or 4th year. This includes core (for both streams) and departmental electives.

(i) It is understood that any 5th semester course can be taken by a student who had mathematics as a pre-major in 2nd year. Thus, these prerequisites are not included in the diagram.

(ii) The courses offered in the 9th and 10th semesters are not included in this diagram. Typically, the electives offered would be specialized for the two streams and the prerequisite would be satisfied by a math major, specializing in the corresponding stream.



Courses in a Nutshell

Semester	Course code	Course name	Type (Theory/Lab)	Category (Core/Elective)	credits L-T-P	Course credits
5	MA3101	Analysis-III	Theory	Core	3-1-0	4
5	MA3102	Algebra-I	Theory	Core	3-1-0	4
5	MA3103	Introduction to Graph Theory and Combinatorics	Theory	Core	3-1-0	4
5	MA3104	Linear Algebra-II	Theory	Core	3-1-0	4
5	MA3105	Numerical Analysis	Theory	Elective	2-0-2	4
6	MA3201	Topology	Theory	Core	3-1-0	4
6	MA3202	Geometry of Curves and Surfaces	Theory	Core (Stream M)	3-1-0	4
6	MA3203	Algebra-II	Theory	Core	3-1-0	4
6	MA3204	Analysis-IV	Theory	Core	3-1-0	4
6	MA3205	Statistics-I	Theory	Core (Stream S)	3-1-0	4
7	MA4101	Algebra-III	Theory	Core	3-1-0	4
7	MA4102	Functional Analysis	Theory	Core	3-1-0	4
7	MA4103	Alnalysis-V	Theory	Core	3-1-0	4
7	MA4104	Algebraic Topology	Theory	Core (Stream M)	3-1-0	4
7	MA4105	Elementary Number Theory	Theory	Elective	3-1-0	4
7	MA4106	Statistics-II	Theory	Elective	2-0-2	4
7	MA4107	Statistical Inference	Theory	Core (Stream S)	3-1-0	4
8	MA4201	Complex Analysis	Theory	Core	3-1-0	4
8	MA4202	Ordinary Differential Equations	Theory	Core	3-1-0	4
8	MA4203	Probability II	Theory	Core	3-1-0	4
8	MA4204	Representation Theory of Finite Groups	Theory	Elective	3-1-0	4
8	MA4205	Differential Geometry	Theory	Core (Stream M)	3-1-0	4
8	MA4206	Linear Models	Theory	Core (Stream S)	3-1-0	4
8	MA4207	Machine Learning & Network Analysis	Theory	Elective	2-0-2	4
8	MA4208	Independent Study I	Theory	Elective		4

Courses in a Nutshell (Contd.)

Semester	Course code	Course name	Type (Theory/Lab)	Category (Core/Elective)	credits L-T-P	Course credits
9	MA5101	MS Project I		Core		8
9	MA5102	Partial Differential Equations	Theory	Core	3-1-0	4
9	MA5103	Fourier Analysis	Theory	Elective	3-1-0	4
9	MA5104	Operator Theory	Theory	Elective	3-1-0	4
9	MA5105	Algebraic Number Theory	Theory	Elective	3-1-0	4
9	MA5106	Topics in Complex Analysis	Theory	Elective	3-1-0	4
9	MA5107	Stochastic Processes	Theory	Elective	3-1-0	4
9	MA5108	Multivariate Statistics	Theory	Elective	3-1-0	4
9	MA5109	Nonparametric Statistics	Theory	Elective	3-1-0	4
9	MA5110	Introduction to Lie Algebra	Theory	Elective	3-1-0	4
9	MA5111	Statistical Decision Theory	Theory	Elective	3-1-0	4
9	MA5112	Theory of Sample Surveys	Theory	Elective	3-1-0	4
9	MA5113	Reliability theory	Theory	Elective	3-1-0	4
9	MA5114	Riemannian Geometry	Theory	Elective	3-1-0	4
9	MA5115	Independent study II	Theory	Elective		4
9	MA5116	Introduction to PDE and Distribution Theory	Theory	Elective	3-1-0	4
9	MA5117	Introduction to Ergodic Theory	Theory	Elective	3-1-0	4
9	MA5118	Commutative Algebra	Theory	Elective	3-1-0	4

Courses in a Nutshell (Contd.)

Semester	Course code	Course name	Type (Theory/Lab)	Category (Core/Elective)	credits L-T-P	Course credits
10	MA5201	MS Project II		Core		12
10	MA5202	Algebraic Geometry	Theory	Elective	3-1-0	4
10	MA5203	Topics in Operator Theory	Theory	Elective	3-1-0	4
10	MA5204	Several Complex Variables	Theory	Elective	3-1-0	4
10	MA5205	Advanced Partial Differential Equations	Theory	Elective	3-1-0	4
10	MA5206	Topics in Analysis	Theory	Elective	3-1-0	4
10	MA5207	Topology and geometry	Theory	Elective	3-1-0	4
10	MA5208	Introduction to Bayesian Analysis	Theory	Elective	3-1-0	4
10	MA5209	Time Series Analysis	Theory	Elective	3-1-0	4
10	MA5210	Advanced Data Structures and Algorithims	Theory	Elective	3-1-0	4
10	MA5211	High Dimensional Statistics	Theory	Elective	3-1-0	4
10	MA5212	Regression Analysis	Theory	Elective	3-1-0	4
10	MA5213	Sobolev Spaces: Theory and Applications	Theory	Elective	3-1-0	4
10	MA5214	Principal Bundles & Representation Ring	Theory	Elective	3-1-0	4
10	MA5215	Analytic Number Theory	Theory	Elective	3-1-0	4
10	MA5216	Independent Study III	Theory	Elective	3-1-0	4
10	MA5217	Independent Study IV	Theory	Elective	3-1-0	4

This is a clickable page and clicking on the semesters leads to the details of the BS-MS courses offered in that particular semester.

Courses offered by the Department of Mathematics & Statistics

DMS Courses

1st Semester

MA1101: Mathematics I **Core course/3 credits**

Part I: Set Theory I

Standard Operations: unions & intersections, various laws, complementation, cartesian product, symmetric difference.

Relations: various relations, equivalence classes, partition.

Mappings: injective, surjective, bijective, inverse of a function, examples, characteristic functions, step functions.

Part II: Number Systems

Construction of natural numbers: may mention Peano's Axioms.

Construction of integers: via equivalence relation on $\mathbb{N} \times \mathbb{N}$.

Construction of rational numbers: via equivalence relation on $\mathbb{Z} \times \mathbb{Z}$.

Part III: Propositional Calculus

Logical quantifiers: examples from set theory.

Negation: contrapositive statements involving various quantifiers.

Part IV: Methods of Mathematical Proof

Mathematical induction: examples including $AM \geq GM$, partial sum of a geometric or arithmetic progression, tower of Hanoi, derangement.

Pigeonhole principle: examples including Dirichlet's Approximation Theorem, arrangement of points on a square or sphere.

Proof by contradiction: examples including infinitude of primes, $\sqrt{2}$ is irrational, if $0 \leq a < \varepsilon$ for every $\varepsilon > 0$ then $a = 0$.

Part V: Theory of Equations

Polynomials: degree of a polynomial, examples.

Properties of roots: definition, repeated roots, Fundamental Theorem of Algebra (statement only), relation between roots and coefficients, number and location of real roots

(Descartes's rule of sign, Sturm's Theorem (statement only)), conjugate root Theorems, roots of cyclotomic polynomials and geometric description.

Methods of solving equations: Cardano's method, Ferrari's method.

Part VI: Calculus

Basic notions: limit, continuity, differentiability, chain rule, Leibniz rule.

Mean Value Theorems: Rolle's Theorem (statement only), Mean Value Theorem, Taylor's Theorem of order 2, L'Hospital's rule.

Applications of derivatives: monotone function, maxima and minima, convex function.

Part VII: Geometry of Curves

Graphing curves: curve tracing, asymptotes.

Tangent & normal: derivative and its geometric/physical meaning.

Geometric notions: radius of curvature, points of inflexion.

Part VIII: Basic Inequalities

Examples including, triangle inequality, p-power inequality, Young's inequality, Cauchy-Schwarz inequality, Hölder's, Jensen's, Minkowski's inequalities, AM-GM-HM.

Suggested Texts:

1. Apostol, T.M., Calculus I, Wiley India Pvt Ltd.
2. Apostol, T.M., Calculus II, Wiley India Pvt Ltd.
3. Artin, M., Algebra, Prentice-Hall of India, 2007.
4. Barnard, S. and Child, J.M., Higher Algebra, Macmillan, 1936.
5. Bartle, R.G., Sherbert, D.R., Introduction to Real Analysis, John Wiley & Sons, 1992.
6. Denlinger, C.G., Elements of Real Analysis, Jones & Bartlett Learning, 2010.
7. Halmos, P.R., Naïve Set Theory, Springer
8. Kreyszig, E., Advanced Engineering Mathematics (8th Edition), Wiley India Pvt Ltd, 2010.
9. Piskunov, N., Differential and Integral Calculus: Volume 1, CBS, 1996.
10. Polya, G., How to Solve It, Princeton University Press, 2004.

2nd Semester

MA1201: Mathematics II **Core course/3 credits**

Part I: Sequences & Series

Sequences: definition, notion of convergence, properties of limit, sandwich Theorem, examples including $1/n$, r^n, $(-1)^n$, $\sin(n)$, n, $(-1)^n n$, $c^{1/n}$ for $c > 0$.

Series: partial sums, definition of convergence, geometric progression, comparison test, telescopic sum, examples including $1/n^p$.

Part II: Linear Algebra in dimensions 2 and 3

Basic material: system of linear equations, elementary operations, linear transformation in $\mathbb{R}^2$ and $\mathbb{R}^3$.

Determinant: definition and its geometric meaning.

Rigid motions in $\mathbb{R}^2$ & $\mathbb{R}^3$: isometry, symmetry of planar objects, distance preserving maps.

Eigenvalues: definition and examples including symmetric matrices.

Conic sections: examples and properties.

Part III: Introduction to Integration

Basic material: definition of integral as a Riemann sum, elementary properties: linearity, order preserving, integral of $|f|$ versus modulus of integral.

Fundamental Theorem of Calculus: statement and proof for continuous functions, computation of $\int_0^x f(t)dt$ for step functions and $f(x) = |x|$.

Further topics: integration by parts, change of variables, Cauchy-Schwarz inequality.

Part IV: Set Theory II

Finite & infinite sets: countable and uncountable sets, Hilbert's infinite hotel, Russell's barber paradox, power set, sets indexed by another set.

Fundamental arguments: Cantor's Theorem, Cantor's diagonalization argument, Schröder-Bernstein Theorem.

Suggested Texts:

1. Apostol, T.M., Calculus I, Wiley India Pvt Ltd.

2. Apostol, T.M., Calculus II, Wiley India Pvt Ltd.
3. Artin, M., Algebra, Prentice-Hall of India, 2007.
4. Barnard, S. and Child, J.M., Higher Algebra, Macmillan, 1936.
5. Bartle, R.G., Sherbert, D.R., Introduction to Real Analysis , John Wiley & Sons, 1992.
6. Denlinger, C.G., Elements of Real Analysis, Jones & Bartlett Learning, 2010.
7. Halmos, P.R., Naïve Set Theory, Springer
8. Kreyszig, E., Advanced Engineering Mathematics (8th Edition), Wiley India Pvt Ltd, 2010.
9. Piskunov, N., Differential and Integral Calculus: Volume 1, CBS, 1996.
10. Polya, G., How to Solve It, Princeton University Press, 2004.

MA1202: Mathematical Methods I **Core course/3 credits**

Part I: Matrix Theory

Liner System: System of linear equations and its solvability, Gauss elimination and Gauss-Jordan elimination methods.

Matrices and Determinants: Rank and inverse of a matrix, some important classes of matrices: Symmetric, anti-symmetric, orthogonal, Hermitian and unitary matrices; determinant of a matrix, similarity transformation.

Eigenvalues: Eigenvalues and eigenvectors; calculation of eigenvalues, Cayley-Hamilton theorem.

Part II: Ordinary Differential Equations

First Order Equations: Liner equations, integrating factor, Bernoulli's Equation, Clairaut's Equation.

Second Order Equations: Liner equations with constant coefficients, general solutions, non-homogenous equations, complementary function and particular integral. Linear equations with variable coefficients- power series solutions.

Part III: Calculus II

Calculus with Complex Numbers: Differentiability- Cauchy-Riemann conditions, analytic functions, singularities. Cauchy's theorem, residue theorem, applications to the calculate of sums and integrals.

Suggested Texts:

1. Apostol, T.M., Calculus I (2nd Edition), Wiley India Pvt Ltd, Springer-Verlag, 2011.

2. Apstol, T.M., Calculus II (2nd Edition), Wiley India Pvt Ltd, Springer-Verlag, 2017.

3. Arfken, G. B., Weber, H. and Harris, F., Essential Mathematical Methods for Physicists and Engineers, Academic Press, 2003.

4. Boas, M. L., Mathematical Methods In the Physical Sciences (3rd Edition), Wiley India Pvt Ltd, Springer-Verlag, 2006.

5. Kreyszig, E., Advanced Engineering Mathematics (8th Edition), Wiley India Pvt Ltd, 2010.

3rd Semester

MA2101: Analysis I **Core course/4 credits**

Real Numbers: Introduction to the real number field, supremum, infimum, completeness axiom, basic properties of real numbers, decimal expansion, construction of real numbers.

Sequences and Series: Convergence of a sequence, Cauchy sequences and subsequences, absolute and conditional convergence of an infinite series, Riemann's theorem, various tests of convergence.

Point-set Topology of $\mathbb{R}$: Open and closed sets; interior, boundary and closure of a set; Bolzano-Weierstrass theorem; sequential definition of compactness and the Heine-Borel theorem.

Limit of a Function: Limit of a function, elementary properties of limits.

Continuity: Continuous functions, elementary properties of continuous functions, intermediate value theorem, uniform continuity, properties of continuous functions defined on compact sets, set of discontinuities.

Suggested Texts:

1. Al-Gwaiz, M.A. and Elsanousi, S.A., Elements of Real Analysis, CRC Press.

2. Apostol, T.M., Mathematical Analysis, Narosa Publishers.

3. Bartle, R.G. and Sherbert, D.R., Introduction to Real Analysis, Wiley.

4. Berberian, S.K., A First Course in Real Analysis, Springer.
5. Denlinger, C.G., Elements of Real Analysis, Jones and Bartlett Publishers.
6. Goldberg, R.R., Methods of Real Analysis, Oxford & IBH Publishing.
7. Howie, J.M., Real Analysis, Springer-Verlag.
8. Kumar, A. and Kumaresan, S., A Basic Course in Real Analysis, CRC Press.
9. Morrey, C.B. and Protter, M.H., A First Course in Real Analysis, Springer-Verlag.
10. Rudin, W., Principles of Mathematical Analysis, McGraw-Hill.

MA2102: Linear Algebra I **Core course/3 credits**

Basic Material: System of linear equations, matrices, elementary row and column operations, echelon forms etc.

Vector Spaces: Definition of a vector space, subspace, quotient space and their examples, linear independence, basis and dimension, rank-nullity theorem.

Matrix and Determinant: Matrices as linear transformations, trace of a matrix, rank of a matrix, determinant of a matrix, properties of determinant, non-singularity, similar matrices, elementary matrices, special types of matrices, change of basis, dual spaces.

Eigenvalues and Eigenvectors: Eigenvalues and eigenvectors of a matrix, characteristic polynomial, diagonalization.

Inner Product Spaces: Orthogonal basis and the Gram-Schmidt orthogonalization process, orthogonal, unitary and Hermitian matrices, similarity and unitary similarity, Schur's triangularization theorem and the spectral theorem for normal matrices (statements only and if time permits)

Suggested Texts:

1. Axler, S., Linear Algebra Done Right, Springer-Verlag.
2. Friedberg, S.H., Insel, A.J. and Spence, L.E., Linear Algebra, Prentice-Hall.
3. Halmos P.R., Finite Dimensional Vector Spaces, Springer.
4. Hoffman, K. and Kunze, R., Linear Algebra, Prentice-Hall.
5. Horn, R. and Johnson, C.R., Matrix Analysis, Cambridge University Press.
6. Kumeresan, S., Linear Algebra: Geometric Approach, Narosa Publishing

7. Lang, S., Introduction to Linear Algebra, Springer-Verlag.

8. Rao, A.R. and Bhimasankaran, P., Linear Algebra, Hindustan Book Agency.

MA2103: Mathematical Methods II **Core course/3 credits**

Prerequisite: Mathematical Methods I (MA1202)

Part I: Partial Differential Equations

Separation of Variables: Classification, solution of partial differential equations by method of separation of variables with special emphasis on the Laplace/Poisson equations.

Part II: Fourier Series

Elementary Introduction to Fourier Series: Fourier coefficients, Fourier series of a function, summation of series using Fourier series.

Part III: Probability and Statistics

Probability: Events, notion of probability, conditional probability, independence, Bayes' theorem, law of large numbers and the central limit theorem (statement only).

Statistics: Mean, median, mode; variance; correlation and regression.

Part IV: Numerical Methods

Root finding by the bisection, regula falsi and Newton-Raphson method; solving ordinary differential equations by the Euler and Runge-Kutta methods; interpolation and extrapolation from data sets.

Suggested Texts:

1. Apostol, T.M., Calculus I (2nd Edition), Wiley India Pvt Ltd, Springer-Verlag, 2011.

2. Apostol, T.M., Calculus II (2nd Edition), Wiley India Pvt Ltd, Springer-Verlag, 2017.

3. Arfken, G. B., Weber, H. and Harris, F., Essential Mathematical Methods for Physicists and Engineers, Academic Press, 2003.

4. Boas, M. L., Mathematical Methods In the Physical Sciences (3rd Edition), Wiley India Pvt Ltd, Springer-Verlag, 2006.

5. Kreyszig, E., Advanced Engineering Mathematics (8th Edition), Wiley India Pvt Ltd, 2010.

4th Semester

MA2201: Analysis II **Core course/4 credits**

Prerequisite: Analysis I (MA2101)

Differentiation: Definition and basic properties, higher order derivatives, Leibnitz's theorem on successive differentiation.

Mean Value Theorems: Rolle's theorem, Lagrange's and Cauchy's mean value theorems, Taylor's theorem, computation of Taylor's series, L'Hôpital's rule

Maxima and Minima: Maxima and minima of a function of one variable, saddle points, applications.

Integration: Riemann integral viewed as an area, partitions, upper and lower integrals, Riemann integrability of a function, basic properties of Riemann integrals, mean value theorems for Riemann integrals, fundamental theorem of integral calculus, change of variable formula and integration by parts, improper Riemann integral, beta and gamma functions.

Sequence of functions: Uniform convergence, convergence and continuity, Weierstrass' approximation theorem.

Suggested Texts:

1. Al-Gwaiz, M.A. and Elsanousi, S.A., Elements of Real Analysis, CRC Press.
2. Apostol, T.M., Calculus I (2nd edition), Narosa Publishers.
3. Bartle, R.G. and Sherbert, D.R., Introduction to Real Analysis, Wiley.
4. Berberian, S.K., A First Course in Real Analysis, Springer.
5. Denlinger, C.G., Elements of Real Analysis, Jones & Bartlett Publishers.
6. Goldberg, R.R., Methods of Real Analysis, Oxford & IBH Publishing.
7. Howie, J.M., Real Analysis, Springer-Verlag.
8. Kumar, A. and Kumaresan, S., A Basic Course in Real Analysis, CRC Press.
9. Morrey, C.B. and Protter, M.H., A First Course in Real Analysis, Springer-Verlag
10. Rudin, W., Principles of Mathematical Analysis (3rd Edition), McGraw-Hill.

MA2202: Probability I **Core course/4 credits**

Prerequisite: Linear Algebra I (MA2102)

Probability: Classical definition, frequency definition and set theoretic definition of probability for discrete sample spaces, basic probability theorems (union of events/Boole's inequality, etc.), independence of events, conditional probability, Bayes' theorem, discrete probability distributions (binomial, Poisson, hypergeometric, geometric, negative binomial), continuous probability distributions (exponential, uniform, normal), moments and moment generating function, basic limit theorems (Chebyshev's inequality, weak law of large numbers, normal approximation of binomial, central limit theorem in iid case), joint distribution of two random variables (with emphasis on the discrete case), notions of conditional expectation and variance, maximum and minimum order statistics.

Markov Chain: Discrete state space, discrete time, Chapman-Kolmogorov equations, classification of states, limiting probabilities.

Suggested Texts:

1. Cacoullos, T., Exercises in Probability, Springer.
2. Feller, W., An Introduction to Probability Theory and Its Applications, Vol. I, Wiley.
3. Gupta, S.C. and Kapoor, V.K., Fundamentals of Mathematical Statistics, S. Chand.
4. Hoel, P.G., Port, S.C. and Stone, C.J., Introduction to Probability Theory, Thomson Brooks/Cole.
5. Ross, S., Introduction to Probability Models, Prentice-Hall.
6. Miller, I. and Miller, M., John E. Freund's Mathematical Statistics with Applications, Pearson.

5th Semester

MA3101: Analysis III **Core course/4 credits**

Prerequisites: Linear Algebra I (MA2102) and Analysis II (MA2201)

Topology in $\mathbb{R}^n$: Open sets, closed sets, compact sets, Heine-Borel theorem, path connectedness in $\mathbb{R}^n$.

Differential Calculus: Directional derivatives and its drawbacks, total derivative, comparison with differentiability on $\mathbb{R}$, chain rule and its applications, C^k functions, mixed derivatives, Taylor's theorem, smooth functions with compact supports, inverse function theorem, implicit function theorem and the rank theorem, examples, maxima and minima, critical point of the Hessian, constrained extrema and Lagrange's multipliers, examples.

Integral Calculus: Line integrals, behaviour of line integral under a change of parameter, independence of path, conditions for a vector field to be a gradient, concept of potential and its construction on convex sets, multiple Riemann integrals, Fubini's theorem (statement only), change of variables (statement only).

Suggested Texts:

1. Apostol, T.M.: Calculus II, Wiley India Pvt. Ltd.
2. Spivak, M.: Calculus on Manifolds, Westview Press
3. Rudin, W.: Principles of Mathematical Analysis, McGraw-Hill

MA3102: Algebra I **Core course/4 credits**

Prerequisite: Mathematics pre-major or instructor's consent

Groups: Definition of groups, subgroups, group homomorphisms and isomorphisms, normal subgroups, quotient groups, Lagrange's theorem, isomorphism theorems, direct sum of abelian groups, direct products, group as symmetries, free group (if time permits).

Group Action: Group actions, conjugacy classes, orbits and stabilizers, class equations.

Symmetric Groups: Symmetric groups, simple groups, simplicity of A_n with $n > 4$.

Sylow's theorems: Three theorems of Sylow, classification of finitely generated abelian groups.

Suggested Texts:

1. Artin, M., Algebra, Prentice-Hall.
2. Dummit, D.S. and Foote, R.M., Abstract Algebra, Wiley.
3. Fraleigh, J.B., A First Course in Abstract Algebra, Narosa Publishers.
4. Gopalakrishnan, N.S., University Algebra, New Age International.
5. Herstein, I.N., Topics in Algebra, Wiley.
6. Hungerford, T.W., Algebra, Springer-Verlag.

7. Malik, D.S., Mordeson, J.M. and Sen, M.K., Fundamentals of Abstract Algebra, McGraw-Hill.

MA3103: Introduction to Graph Theory & Combinatorics
Core course/4 credits

Prerequisite: Linear Algebra I (MA2102)

Graph Theory: Fundamental concepts and basic definitions, path, cycles and trees, graph isomorphism, Eulerian and Hamiltonian graphs, planarity, connectivity, graph colourings, Euler's formula for planar graphs and proof of 5 color theorem, matching, spectral graph theory; Numbered trees and Prufer code (if time permits).

Combinatorics: Inclusion-exclusion principle, pigeon hole principle; Sperner's theorem, Hall marriage theorem; Mantel's theorem, Turan's theorem; graphic sequences (if time permits).

Suggested Texts:

1. Bollobas, B., Modern Graph Theory, Springer-Verlag.
2. Godsil, C. and Royle, G., Algebraic Graph Theory, Springer-Verlag.
3. West, D., Introduction to Graph Theory, Prentice-Hall.

MA3104: Linear Algebra II **Core course/4 credits**

Prerequisite: Linear Algebra I (MA2102)

Eigenvalues: Annihilating polynomials, minimal polynomials, invariant subspaces, simultaneous diagonalization and triangularization, direct sum decompositions, primary decomposition theorem.

Canonical Forms: Cyclic decompositions and the rational form, Jordan canonical form, computation of invariant factors.

Matrix Decomposition: Orthogonal, unitary and Hermitian matrices, unitary similarity, Schur's triangularization theorem, spectral theorem for normal matrices, positive definite matrices, polar decomposition, singular value decomposition.

Bilinearity: Bilinear maps, tensors and tensor product with emphasize on inner product space.

Exterior Forms: Quadratic forms, bilinear forms, symmetric and alternating forms, determinant function and uniqueness of determinant.

Suggested Texts:

1. Axler, S., Linear Algebra Done Right, Springer-Verlag.
2. Friedberg, S.H., Insel, A.J. and Spence, L.E., Linear Algebra, Prentice-Hall.
3. Horn, R. and Johnson, C.R., Matrix Analysis, Cambridge University Press.
4. Hoffman, K. and Kunze, R., Linear Algebra (2nd Edition), Prentice-Hall.
5. Lang, S., Introduction to Linear Algebra (2nd Edition), Springer-Verlag.
6. Lax, P.D., Linear Algebra and Its Applications, Wiley.
7. Rao, A.R. and Bhimasankaran, P., Linear Algebra (2nd Edition), Hindustan Book Agency.
8. Roman, S., Advanced Linear Algebra, Springer.
9. Shafarevich, I.R. and Remizov, A.O., Linear Algebra and Geometry, Springer.

MA3105: Numerical Analysis **Elective course/4 credits**
This course will have 2 theory hours and 2 lab hours

Prerequisites: Linear Algebra I (MA2102) and Analysis II (MA2201)

Interpolation: Newton's forward, backward and divided difference formulae; Lagrange's method; Gauss, Stirling and Bessel's formulae; spline interpolation.

Solution of Systems of Linear Equations: Gauss elimination method, Thomas algorithm, Gauss-Jacobi and Gauss-Seidel methods.

Eigenvalues and Eigenvectors: Power method, Jacobi's method, Given's method, Householder's method.

Solution of Non-linear Equations: Bisection method, fixed point iteration, Newton-Raphson, secant and regula-falsi methods.

Solution of System of Non-linear equations: Fixed-point method and Newton's method.

Numerical Differentiation: Formulae based on Newton's forward, backward, divided difference, and Lagrange's formulae.

Numerical Integration: Trapezoidal rule, Simpson's 1/3rd and 3/8th rules, Weddle's rule, quadrature formulae based on Stirling's and Bessel's interpolation formulae.

Solution of Differential Equations: Taylor's series method, Picard's method and Runge-Kutta methods (1st, 2nd and 4th orders) for solving ODEs.

Suggested Texts:

1. Atkinson, K., Elementary Numerical Analysis, Wiley.

2. Burden, R.L. and Faires, J.D., Numerical Analysis, Brooks/Cole, International edition.
3. Conte, S.D. and De Boor, C., Elementary Numerical Analysis: An Algorithmic Approach, Tata McGraw-Hill.
4. Froberg, C.E., Introduction to Numerical Analysis, Addison-Wesley.
5. Scarborough, J.B., Numerical Mathematical Analysis, Johns Hopkins University Press.

6th Semester

MA3201: Topology **Core course/4 credits**

Prerequisite: Analysis III (MA3101).

Metric Spaces: Metric space topology, equivalent metrics, sequences, complete metric spaces, limits and continuity, uniform continuity, extension of uniformly continuous functions.

Topological Spaces: Definition, examples, bases, sub-bases, product topology, subspace topology, metric topology, quotient topology, second countability and separability.

Continuity: Continuous functions on topological spaces, homeomorphisms.

Connectedness: Definition, example, path connectedness and local connectedness.

Compactness: Definition, limit point compactness, sequential compactness, net and directed set, local compactness, Tychonoff theorem, Stone-Weierstrass theorem, Arzela-Ascoli theorem.

Separation Axioms: Hausdorff, regular and normal spaces; Urysohn lemma and Tietze extension theorem; compactification.

Metrizability: Urysohn metrization theorem.

Suggested Texts:

1. Armstrong, M.A., Basic Topology, Springer-Verlag.
2. Dugundji, J., Topology, Allyn and Bacon Series in Advanced Mathematics, Allyn & Bacon.
3. Kelley, J.L., General Topology, Springer-Verlag.
4. Munkres, J.R., Topology (2nd Edition), Prentice-Hall.

5. Simmons, G.F., Introduction to Topology and Modern Analysis, Tata McGraw-Hill.

MA3202: Geometry of Curves and Surfaces
Core course (for Stream M)/4 credits

Prerequisite: Analysis III (MA3101).

Part I : Curves

Curves: Parametrized and regular curves, arc length, parametrization by arc length.

Local Theory: Tangent-normal-binormal frame, curvature, torsion, fundamental theorems of local theory of plane and space curves.

Global Theory: Simple curves, Jordan curve theorem (without proof), isoperimetric inequality, four-vertex theorem.

Part II : Surfaces

Surfaces: Parametrization, change of parameters, smooth functions, tangent plane, differential, diffeomorphism, inverse and implicit function theorems.

Second Fundamental Form and Curvature: Gauss map; oriented surfaces; second fundamental form; Gauss, mean and principal curvatures; normal sections.

Integration on Surface: Definition of integral, partitions of unity, change of variables formula, divergence theorem.

Geometry of Surfaces: Rigid motions and isometries, Gauss's Theorema Egregium, geodesics.

Gauss-Bonnet Theorem : Index of a vector field at an isolated zero, Euler characteristic (if time permits).

Suggested Texts:

1. Berger, M. and Gostiaux, B., Differential Geometry: Manifolds, Curves and Surfaces, Springer-Verlag.
2. Do Carmo, M.P., Differential Geometry of Curves and Surfaces, Prentice-Hall.
3. Montiel, S. and Ros, A., Curves and Surfaces, Graduate Studies in Mathematics, Vol. 69, American Mathematical Society.
4. O'Neill, B., Elementary Differential Geometry (2nd Edition), Academic Press.
5. Pressley, A., Elementary Differential Geometry, Springer-Verlag.
6. Thorpe, J. A., Elementary Topics in Differential Geometry, Springer-Verlag.

MA3203: Algebra II **Core course/4 credits**

Prerequisite: Algebra I (MA3102).

Rings and Ideals: Rings and ring homomorphism, ideals, quotient rings, zero-divisors, units, prime and maximal ideals, nilradical and Jacobson radical operations on ideals, extension and contraction, division in domains, g.c.d. and l.c.m., division algorithm, Euclidean domain, unique factorization domain, principal ideal domain.

Modules: Modules and module homomorphisms, submodule and quotient modules, operations on submodules, direct sum and product, finitely generated modules, classification of finitely generated modules over PIDs, exact sequences of modules, tensor product of modules, canonical forms.

Suggested Texts:

1. Artin, M., Algebra, Prentice-Hall.
2. Atiyah, M.F. and MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley.
3. Dummit, D.S. and Foote, R.M., Abstract Algebra, Wiley.
4. Eisenbud, D., Commutative Algebra with a view towards Algebraic Geometry, Springer-Verlag.
5. Gopalakrishnan, N.S., Commutative Algebra, Oxonian Press.
6. Kunz, E., Introduction to Commutative Algebra and Algebraic Geometry, Birkhäuser.
7. Luthar, I.S. and Passi, I.B.S., Algebra, Vol. 2: Rings, Narosa Publishing House.
8. Luthar, I.S. and Passi, I.B.S., Algebra, Vol. 3: Modules, Narosa Publishing House.
9. Matsumura, H., Commutative Ring Theory, Cambridge University Press.
10. Reid, M., Undergraduate Commutative Algebra, London Mathematical Society Student Texts (29), Cambridge University Press.
11. Sharp, R.Y., Steps in Commutative Algebra, London Mathematical Society Student Texts (19), Cambridge University Press.

MA3204: Analysis IV **Core course/4 credits**

Prerequisite: Analysis III (MA3101).

Introduction: Drawbacks of Riemann integration, measurement of length– introductory remarks.

Abstract Measures: Algebra, σ-algebra and Borel σ-algebra, outer measure, measure, Caratheodory extension Theorem and construction of Lebesgue measure on $\mathbb{R}^n$ as an application, measure space, measurable set and measurable function.

Integration Theory: Definition and properties of Lebesgue integral, basic convergence theorems– monotone convergence theorem, Fatou's lemma and dominated convergence theorem.

Borel Measure: Regularity properties of Borel measure, Radon measure, Caratheodory's criterion; Continuity properties of measurable functions– Lusin's and Egoroff's theorems.

L^p Spaces: Fundamental inequalities - Hölder's inequality, Jensen's inequality and Minkowski's inequality, definition of L^p spaces, completeness, approximation by continuous functions.

Product Measure: Measurability in product spaces, product measures, Fubini and Fubini-Tonelli theorems, polar coordinates and change of variable theorem.

Suggested Texts:

1. De Barra, G., Measure Theory and Integration, New Age International Publishers.

2. Evans, L.C. and Gariepy, R.F., Measure Theory and Fine Properties of Functions, CRC Press.

3. Folland, G.B., Real Analysis: Modern Techniques and Their Applications (2nd Edition), Wiley-Interscience.

4. Kantorovitz, S., Introduction to Modern Analysis, Oxford University Press.

5. Rana, I.K., An Introduction to Measure and Integration, Narosa Publishers.

6. Royden, H.L., Real Analysis, Prentice-Hall.

7. Rudin, W., Real and Complex Analysis, McGraw-Hill.

MA3205: Statistics I **Core course (for Stream S)/4 credits**

Prerequisite: Probability I (MA2202)

Descriptive Statistics: Data, frequency distribution, line diagram, bar diagram, histogram, pie chart, pictorial presentation of data.

Applications of Different Moments: Measures of central tendency and dispersion, moments, Sheppard's correction, measures of skewness and kurtosis.

Correlation and Regression: Product moment correlation and rank correlation, simple regression, multiple regression, multiple correlation and partial correlation coefficients, forecasting.

Sampling: Drawing of samples based on SRSWR and SRSWOR, drawing of samples from different discrete and continuous statistical distributions.

Inference: Point estimation and interval estimation, tests of normal population mean and population variance, tests for equality of population means, tests for equality of population variances, paired-t test, test for population correlation coefficient, large sample tests for population mean, χ^2 test for goodness of fit, χ^2 test for independence of attributes.

Suggested Texts:

1. Goon, A.M., Gupta, M. and Dasgupta, B., Fundamentals of Statistics, Vols. I & II, World Press.

2. Miller, I. and Miller, M., John E. Freund's Mathematical Statistics with Applications, Pearson.

3. Rencher, A.C., Methods of Multivariate Analysis, Wiley.

4. Snedecor, G.W. and Cochran, W.G., Statistical Methods. Iowa State University Press.

7th Semester

MA4101: Algebra III **Core course/4 credits**

Prerequisite: Algebra II (MA3203).

Fields: Fields, field of fractions, field extensions, algebraic extensions, degree of an extension, splitting fields, normal extensions, separable extensions, finite fields.

Galois Theory: Galois extensions, automorphism groups and fixed fields, fundamental theorem of Galois theory and applications, cyclic extensions, cyclotomic polynomials, solvable groups, solvability by radicals, constructibility of regular polygons, transcendental extensions.

Suggested Texts:

1. Artin, M., Algebra, Prentice-Hall.
2. Artin, E., Algebra with Galois Theory, Courant Lecture Notes.
3. Gopalkrishnan, N.S., University Algebra, New Age International Press.
4. Lang, S., Algebra, Springer.
5. Morandi, P., Field and Galois Theory, Springer-Verlag.

MA4102: Functional Analysis **Core course/4 credits**

Prerequisites: Analysis IV (MA3204) and Topology (MA3201).

Normed Linear Spaces: Definitions, Banach spaces, Hilbert spaces, non-compactness of the unit ball in infinite dimensional normed linear spaces, quotient spaces.

Linear Maps: Boundedness and continuity, linear functionals, isometries.

Convexity: Hahn-Banach extension theorem, complex Hahn-Banach theorem, separation of convex sets, applications.

Completeness: Baire category theorem, Banach-Steinhaus theorem, open mapping theorem and closed graph theorem.

Duality: Dual spaces, Riesz representation theorem, reflexivity, weak topologies, weak convergence, weak compactness, Banach-Alaoglu theorem, adjoints and compact operators with examples, Volterra operators.

Hilbert Spaces: Bessel's inequality, complete systems, Gram-Schmidt orthogonalization, Parseval's identity, projections, orthogonal decomposition, bounded linear functionals in Hilbert spaces.

Spectral Theory: Spectrum, spectral theory of compact self-adjoint operators, Spectral theory of compact normal operators.

Suggested Texts:

1. Brezis, H., Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.

2. Bollabs, B., Linear Analysis: An Introductory Course, Cambridge University Press.

3. Conway, J.B., A Course in Functional Analysis (2nd Edition), Springer-Verlag.

4. Eidelman, Y., Milman, V. and Tsolomitis, A., Functional Analysis: An Introduction, American Mathematical Society.

5. Kesavan, S., Functional Analysis, Hindustan Book Agency.

6. Lax, P.D., Functional Analysis, Wiley-Interscience.

7. Limaye, B.V., Functional Analysis, New Age Publishers.

8. Rudin, W., Functional Analysis, McGraw-Hill.

9. Simmons, G.F., Topology and Modern Analysis, Tata McGraw-Hill.

MA4103: Analysis V **Core course/4 credits**

Prerequisite: Analysis IV (MA3204); Results proved in Functional Analysis (MA4102) course will be useful in the 'Fourier Series' part.

Signed Measures: Total variation measure, absolute continuity, Lebesgue decomposition, Radon-Nikodym theorem, Hahn decomposition theorem.

Convolution: Definition and basic properties, Young's inequality, mollifiers and approximation by smooth functions.

Differentiation Theory: Hardy-Littlewood maximal functions, Lebesgue differentiation theorem, Lebesgue points, absolutely continuous functions, fundamental theorem of calculus, Rademacher theorem.

Fourier Series: Fourier coefficients and series, summability, pointwise convergence of Fourier series, convergence of Fourier series in norm.

Suggested Texts:

1. Duoandikoetxea, J., Fourier Analysis, American Mathematical Society.

2. Evans, L.C. and Gariepy, R.F., Measure Theory and Fine Properties of Functions, CRC Press.

3. Folland, G.B., Real Analysis: Modern Techniques and Their Applications (2nd Edition), Wiley-Interscience.

4. Grafakos, L., Classical Fourier Analysis, Springer-Verlag.

5. Katznelson, Y., An Introduction to Harmonic Analysis, Cambridge University Press.

6. Rudin, W., Real and Complex Analysis, McGraw-Hill.

MA4104: Algebraic Topology **Core course (for Stream M)/4 credits**

Prerequisites: Algebra I (MA3102) and Topology (MA3201).

Fundamental Group: Review of quotient topology, path homotopy, definition of fundamental group, covering spaces, path and homotopy lifting, fundamental group of S^1, deformation retraction, Brouwer's fixed point theorem, Borsuk-Ulam theorem, van-Kampen's theorem, fundamental group of surfaces, universal covering space, correspondence between covering spaces and subgroups of fundamental group.

Homology Theory: Simplicial complexes and maps, homology groups, computation for surfaces.

Suggested Texts:

1. Hatcher, A., Algebraic Topology, Cambridge University Press.
2. Massey, W.S., A Basic Course in Algebraic Topology, Springer-Verlag.
3. Munkres, J.R., Elements of Algebraic Topology, Addison-Wesley.
4. Spanier, E.H., Algebraic Topology, Springer-Verlag.

MA4105: Elementary Number Theory **Elective course/4 credits**

Prerequisite: Consent of the Instructor.

Fundamental Notions: Mathematical induction, divisibility, greatest common divisor and Euclidean algorithm, prime numbers and unique farctorization.

Arithmetic Functions: Multiplicative functions, Mobius function and Mobius inversion, Eulers totient function, greatest integer function, average orders of arithmetic functions.

Congruences: Definition and basic properties, congruence powers and Eulers theorem, linear congruence equations, simultaneous linear equations and Chinese remainder theorem, polynomial congruences, order and primitive roots.

Quadratic Residues and the Quadratic Reciprocity Law: Quadratic residues, Legendre symbol, Gauss lemma, Quadratic reciprocity law, Jacobi symbol.

Sums of Squares: Pythagorean triplet, representations of integers as sums of two squares and sums of four squares.

Binary Quadratic Forms: Introduction to binary quadratic forms, equivalence, reduction of binary quadratic forms and class number, representations of numbers by binary quadratic forms.

Continued Fractions and Pells Equation: Finite and infinite continued fractions, purely periodic continued fractions, Continued fraction expansion of square roots of positive numbers, Pells equation and solutions.

Cryptography: Block and stream ciphers, public key cryptosystems using RSA, El Gamal cryptosystem.

Suggested Texts:

1. Burton, D, M., Elementary Number Theory (6th Edition), Tata McGraw-Hill, 2007.
2. Koshy, T., Elementary Number Theory with Applications (2nd Edition), Academic Press, 2007.
3. Le Veque, W, J., Topics in Number Theory, Vols. I & II, Dover Publications, 2002.
4. Rosen, K, H., Elementary Number Theory and Its Applications (5th Edition), Addison-Wesley, 2000.

MA4106: Statistics II **Elective course/4 credits**

This is a course with significant lab component.

Prerequisite: Statistics I (MA3205).

Simulation: Simulation technique for non-standard distributions including χ^2, t and F distributions (central as well as non-central).

Inference: MLE, drawbacks of method of moment estimation, EM algorithm, Estimation of linear model (with multicollinearity and multiple regression), exact tests for discrete and continuous (specially non-normal) distributions, interval estimation, χ^2 and KS goodness-of-fit test, frequency χ^2 and its use in testing of hypothesis (contingency table), test for ρ.

Suggested Texts:

1. Casella, G. and Berger, R.L., Statistical Inference, Thomson Brooks/Cole.
2. Friedman, J., Hastie, T. and Tibshirani, R., The Elements of Statistical Learning, Springer.
3. Kundu, D. and Basu, A., Statistical Computing: Existing Results and Recent Trends, Narosa Publishing House.
4. Ross, S.M., Introduction to Probability Models, Elsevier.

MA4107: Statistical Inference **Core course (for Stream S)/4 credits**

Prerequisites: Analysis IV (MA3204) and Statistics I (MA3205).

Common Families of Distributions: Location and scale families, exponential families.

Principles of Data Reduction: Introduction, sufficient statistics, minimal sufficient statistics, ancillary statistics, complete statistics, relation between ancillary and complete sufficient statistics.

Point Estimation: Introduction, maximum likelihood estimators, EM algorithm; Methods of evaluating estimators– mean squared error, uniformly minimum variance unbiased estimator (UMVUE), sufficiency and unbiasedness, loss function optimality, asymptotic evaluations, consistency, efficiency.

Hypothesis Testing: Introduction; Methods of finding tests– likelihood ratio tests, Bayesian tests, union-intersection and intersection-union tests, Methods of evaluating tests– error probabilities and the power function, most powerful tests, sizes of union-intersection and intersection-union tests, p-values, loss function optimality.

Interval Estimation: Introduction; Methods of finding interval estimators– pivotal quantities, pivoting the CDF; Methods of evaluating interval estimators– size and coverage probability, test-related optimality, loss function optimality.

Suggested Texts:

1. Casella, G. and Berger, R.L., Statistical Inference, Thomson Brooks/Cole.
2. Cramer, H., Mathematical Methods of Statistics, Princeton University Press.
3. Lehman, E.L. and Casella, G., Theory of Point Estimation, Springer.
4. Lehman, E.L. and Romano, J.P., Testing Statistical Hypotheses, Springer.
5. Rao, C.R., Linear Statistical Inference and Its Applications, Wiley-Interscience.
6. Rohatgi, V.K., Statistical Inference, Dover Publications.
7. Wilks, S.S., Mathematical Statistics, Buck Press.

8th Semester

MA4201: Complex Analysis **Core course/4 credits**

Prerequisites: Analysis III (MA3101) and Topology (MA3201).

Complex Number System: Field of complex numbers, polar representations, power, roots, complex exponential, complex logarithm, extended complex plane, Riemann sphere and stereographic projection.

Analytic Functions: Definitions, Cauchy-Riemann equations, harmonic functions.

Complex Integration: Riemann-Stieltjes integration, power series representation of analytic functions, zeros of an analytic function, winding number, Cauchy's integral formula, Cauchy estimates and Liouville theorem, Cauchy's theorem, Morera's theorem, open mapping theorem, maximum modulus theorem, Schwarz's lemma.

Singularities: Classification of singularities, Laurent series, Casorati-Weierstrass theorem, residues, evaluation of definite integrals using residue theorem, meromorphic functions, argument principle, Rouché's theorem.

Conformal Mappings: Definitions, conformal maps and geometry of Möbius transformations, normality and compactness, Riemann mapping theorem.

Gamma Functions: (If time permits) Gamma functions through functional equations, elementary properties.

Suggested Texts:

1. Ahlfors, L.V., Complex Analysis, McGraw-Hill.
2. Conway, J.B., Functions of One Complex Variable, Springer-Verlag.
3. Gamelin, T.W., Complex Analysis, Springer.
4. Greene, R.E. and Krantz, S.G., Function Theory of One Complex Variable, American Mathematical Society.
5. Lang, S., Complex Analysis, Springer-Verlag.
6. Marsden, J.E., Basic Complex Analysis, W.H. Freeman & Co.
7. Narasimhan, R., Complex Analysis in One Variable, Birkhäuser-Verlag.
8. Rao, M., Stetkae, H. and Fournais, S., Complex Analysis : An Invitation, World Scientific.
9. Rudin, W., Real and Complex Analysis, McGraw-Hill.
10. Stein, E. M. and Shakarchi, R., Complex Analysis, Princeton Lectures in Analysis.

MA4202: Ordinary Differential Equations **Core course/4 credits**

Prerequisite: Analysis III (MA3101).

Fundamental Theory: Existence of solutions under continuity, existence and uniqueness under Lipschitz condition, non-uniqueness and Kneser's theorem, extension of solutions, dependence of solutions with respect to initial data and parameter, flow of an ordinary differential equation.

Boundary-Value Problems of Linear Differential Equations of the Second Order: Zeros of solutions, Sturm-Liouville problems, eigenvalue problems, eigenfunction expansions.

Linear System: Exponentials of operators, fundamental theorem for linear systems, linear systems in $\mathbb{R}^2$.

Stability Theory: Stable, unstable and asymptotically stable points; Liapunov functions; Stable manifolds.

Poincaré-Bendixson Theory: Limit sets, local sections, Poincaré-Bendixson theorem and its applications (if time permits).

Suggested Texts:

1. Barreira. L. and Valls, C., Ordinary Differential Equations: Qualitative Theory, American Mathematical Society .

2. Birkhoff. G. and Rota. G.C., Ordinary Differential Equations, Wiley.

3. Coddington, E.A. and Levinson, N., Theory of Ordinary Differential Equations, McGraw-Hill.

4. Hirsch, M.W., Smale, S. and Devaney, R.L., Differential Equations, Dynamical Systems, and an Introduction to Chaos, Elsevier/Academic Press, Amsterdam.

5. Hsieh, P. and Sibuya, Y., Basic Theory of Ordinary Differential Equations, Springer-Verlag.

6. Perko, L., Differential Equations and Dynamical Systems, Springer-Verlag.

7. Simmons. G.F., Differentials Equations with Applications and Historical Notes, Tata McGraw-Hill.

8. Teschl. G., Ordinary Differential Equations and Dynamical Systems, American Mathematical Society.

MA4203: Probability II **Core course/4 credits**

Prerequisite: Functional Analysis (MA4102).

Quick review of concepts and results (without proof) from basic discrete and continuous random variables, 1-1 correspondence between distribution functions and probabilities on $\mathbb{R}$, examples of probability measures in Euclidean space, a metric on the space of probability measures on $\mathbb{R}^d$, expectation and the convergence theorems, independence, Borel-Cantelli lemma, weak and strong laws in the *iid* cases, Kolmogorov's 0-1 law and three-series theorem, various modes of convergence, infinite products, Kolmogorov's consistency theorem, characteristic functions, uniqueness, inversion theorem, Levy continuity theorem, proof of CLT for the *iid* case with finite variance, martingales, infinitely divisible laws and stable laws.

Suggested Texts:

1. Athreya, K.B. and Lahiri, S.N., Measure Theory and Probability Theory, Springer.
2. Ash, R. and Dolans-Dade, C.A., Probability and Measure Theory, Academic Press
3. Billingsley, P., Probability and Measure, John Wiley.
4. Borkar, V.S., Probability Theory : An Advanced Course, Springer.
5. Chung, K.L., A Course in Probability Theory, Elsevier.
6. Durrett, R., Probability : Theory and Examples, Cambridge University Press.
7. Gut, A., Probability : A Graduate Course, Springer.
8. Loève, M., Probability Theory, Vols. I & II, Springer.
9. Parthasarathy, K.R., Introduction to Probability and Measure, Hindustan Book Agency.

MA4204: Representation Theory of Finite Groups
Elective course/4 credits

Prerequisites: Linear Algebra II (MA3104) and Algebra II (MA3203).

Representations of Finite Groups: Definitions, Schur's theorem, characters, group algebra, Maschke's theorem, simple modules over group algebras, inner products of characters, the number of irreducible characters, character tables and orthogonality relations, Burnside's two-prime theorem, Induced representation, Frobenius reciprocity, construction of character tables of $GL(2, k)$, $SL(2, k)$, $PGL(2, k)$ where k is a finite field.

Further Topics: Brauer's theorem on induced characters, representation of symmetric groups, Young diagrams and Frobenius' character formula.

Suggested Texts:

1. Fulton, W. and Harris, J., Representation Theory : A First Course, Springer-Verlag.
2. James, G. and Liebeck, M., Representations and Characters of Groups, Cambridge University Press.

3. Musili, C., Representations of Finite Groups, Hindustan Book Agency.

4. Serre, J.P., Linear Representation of Finite Groups, Springer.

MA4205: Differential Geometry **Core course (for Stream M)/4 credits**

Prerequisites: Topology (MA3201) and Geometry of Curves & Surfaces (MA3202).

Basic Theory: Topological manifolds, examples, differentiable manifolds and maps, immersed and imbedded manifolds, submanifolds, partitions of unity, compact manifolds as closed submanifolds of $\mathbb{R}^n$.

Tangent Space and Vector Fields: Definition of tangent vector as equivalence class of curves and derivations, tangent spaces and their mappings, tangent bundle, vector fields, integral curves, complete vector fields, Lie derivative and connection with Lie bracket of vector fields.

Differential Forms and Integration: Wedge product, Exterior differentiation: definition, axiomatic treatment and coordinate invariance, closed and exact forms, Poincaré lemma, review of classical line and surface integrals, integration on manifolds, orientation, Stokes' theorem, integration by parts.

Suggested Texts:

1. Guillemin, V. and Pollack, A., Differential Topology, AMS Chelsea.

2. Hirsch, M.W., Differential Topology, Springer.

3. Kumaresan, S., A Course in Differential Geometry and Lie Groups, Hindustan Book Agency.

4. Lee, J.M., Introduction to Smooth Manifolds, Springer-Verlag.

5. Milnor, J,W., Topology from the Differentiable Viewpoint, Princeton University Press.

6. Mukherjee, A., Topics in Differential Topology, Hindustan Book Agency.

7. Spivak, M., A comprehensive Introduction to Differential Geometry, Vol. I, 3rd Edition, Publish or Perish.

8. Tu, L.W., An Introduction to Manifolds, Universitext, Springer-Verlag.

MA4206 : Linear Models **Core course (for Stream S)/4 credits**

Prerequisite: Statistical Inference (MA4107).

Estimation in Linear Model: Linear statistical models, illustrations, normal equations and least squares estimators, g-inverse and solution of normal equations, estimability of linear parametric function, Gauss-Markov theorem, error space and estimation space, variances and covariances of BLUEs, estimation of error variance, Fisher-Cochran theorem, distribution of quadratic forms, fundamental theorems of least squares and applications to tests of linear hypotheses, estimation subject to linear restrictions.

Generalized Linear Model: Logistic regression, log-linear models, link function, inference for general linear model.

Suggested Texts:

1. Bapat, R.B., Linear Algebra and Linear Models, Springer.
2. Kshirsagar, A.M., A Course in Linear Models, Marcel Dekker.
3. Montgomery, D.C., Peck, E.A. and Vining, G.G., Introduction to Linear Regression Analysis, Wiley.
4. Rao, C.R., Linear Statistical Inference and Its Applications, Wiley.
5. Ryan, T.P., Modern Regression Methods, Wiley- Blackwell.
6. Searle, S.R., Linear Models, Wiley Classic Library, CBS Publishers.
7. Sengupta, D. and Jammalamadaka, S.R., Linear Models : An Integrated Approach, World Scientific.

MA4207 : Machine Learning & Network Analysis
Elective course/4 credits

This course has substantial lab component.

Prerequisites: Introduction to Graph Theory & Combinatorics (MA3103) and Statistics I (MA3205).

Machine Learning: Introduction to machine learning, overview of machine learning and basic concepts, feature selection and extraction, dimensionality reduction, small sample size problem, discriminant analysis, nearest neighbour, linear and quadratic classifiers, naïve Bayes' classifiers, support vector machine, clustering, EM algorithm, validation and bootstrapping, cross validation, bias-variance trade off.

Network Analysis: Networks and their representation– weighted-unweighted and directed-undirected networks, hypergraphs; Network structure– degree, path, components; Measures– degree centrality, closeness centrality, betweenness centrality, clustering coefficient, transitivity; Large-scale structure– components, shortest path and small-world property, degree distributions, power law degree distribution, assortative mixing and modularity, network motifs; Network models– random graph, small-world network, scale-free network, duplication and divergence model; Epidemics on networks– SI model, SIR model, SIS model.

Suggested Texts:

1. Bishop, C.M., Pattern Recognition and Machine Learning, Springer.
2. Duda, R.O., Hart P.E. and Stork D.G., Pattern Classification, Wiley-Interscience. 2nd Edition.
3. Fukunaga K., Introduction to Statistical Pattern Recognition, Academic Press.
4. Newman, M.J., Networks: An Introduction, Oxford University Press.

MA4208: Independent Study I **Elective course/4 credits**

Prerequisite: Consent of the Instructor & mentor.

Refer to the guidelines for any independent study course (for 4th year or 5th year BS-MS students), given at the end of this document.

9th Semester

MA5101: MS Project I **Core course/8 credits**

See the end of the document for detailed guidelines.

MA5102: Partial Differential Equations **Core course/4 credits**

Prerequisites: Functional Analysis (MA4102), Analysis V (MA4103) and Ordinary Differential Equations (MA4202).

First-order Equations: Method of characteristics and existence of local solutions.

Characteristic Manifolds and Cauchy Problem: Non-characteristic surfaces, Cauchy-Kowalevski theorem and uniqueness theorem of Holmgren.

Laplace Equation: Fundamental solution, harmonic function and its properties, Poisson's equation, Dirichlet problem and Green's function, existence of solution of the Dirichlet problem using Perron's method, introduction to variational method.

Heat Equation: Fundamental solution and initial-value problem, mean value formula, maximum principle, uniqueness and regularity, nonnegative solutions, Fourier transform methods.

Wave Equation: d'Alembert's formula, method of spherical means, Hadamard's method of descent, Dumahel's principle and Cauchy problem, initial-boundary-value problem, Fourier transform methods.

Suggested Texts:

1. Evans, L.C., Partial Differential Equations, American Mathematical Society.
2. Folland, G., Introduction to Partial Differential Equations, Princeton University Press.
3. Gilbarg, D. and Trudinger, N., Elliptic Partial Differential Equations of Second Order, Springer-Verlag.
4. Han, Q., A Basic Course in Partial Differential Equations, American Mathematical Society.
5. John, F., Partial Differential Equations, Springer-Verlag.
6. McOwen, R.C., Partial Differential Equations: Methods and Applications, Pearson Education.
7. Renardy, M. and Rogers, R., An Introduction to Partial Differential Equations, Springer-Verlag.

MA5103: Fourier Analysis **Elective course/4 credits**

Prerequisites: Analysis IV (MA3204) and Functional Analysis (MA4102).

Fourier Transform: Fourier transform of L^1 functions, Schwartz class functions and L^2 theory.

Interpolation: Riesz-Thorin and Marcinkiewicz interpolations, Hausdorff-Young inequality.

Poisson Integral: Non-tangential convergence, characterization of Poisson integral.

Distributions: Definition and properties, Paley-Weiner Theorem, tempered distributions.

Singular Integrals: Riesz and Bessel potential, Hilbert transform, Calderon-Zygmund decomposition.

Suggested Texts:

1. Duoandikoetxea, J., Fourier Analysis, American Mathematical Society.
2. Grafakos, L., Classical Fourier Analysis, Springer-Verlag.

3. Katznelson, Y., An Introduction to Harmonic Analysis, Cambridge University Press.

4. Körner, T.W., Fourier Analysis, Cambridge University Press.

5. Stein, E.M., Singular Integrals and Differentiability Properties of Functions, Princeton University Press.

6. Stein, E.M. and Weiss, G., Introduction to Fourier Analysis on Euclidean Spaces, Princeton University Press.

MA5104: Operator Theory **Elective course/4 credits**

Prerequisites: Complex Analysis (MA4201) and Functional Analysis (MA4102).

Operators on Hilbert Spaces: Projections and subspaces, commutative C*-algebras, spectral theorem, continuous functional calculus, square root of positive operator, unilateral shift C*-algebras, noncommutative states and representations, Gelfand-Neumark representation theorem.

Banach Algebras: Banach algebras, spectral radius, maximal ideal space, Gelfand transform.

Spectral Theory of Normal Operators: Von-Neumann algebras, projections, double commutant theorem, L^∞ functional calculus.

If time permits, one may choose from the following:

Fredholm Theory: Compact operators on Hilbert Spaces, Fredholm theory, index.

Unbounded Operators: Unbounded operators, domains, graphs, adjoints, spectral theorem.

Suggested Texts:

1. Arveson, W., An Invitation to C*-algebras, Springer-Verlag.

2. Arveson, W., A Short Course in Spectral Theory, Springer-Verlag.

3. Conway, J.B., A Course in Operator Theory, American Mathematical Society.

4. Davidson, K., C*-algebras by Example, Fields Institute Monograph, AMS.

5. Douglas, R.G., Banach Algebra Techniques in Operator Theory, Springer.

6. Kadison, R.V. and Ringrose, J.R., Fundamentals of the Theory of Operator Algebras Vol. I. Elementary Theory, American Mathematical Society.

7. Murphy, G., C*-algebras and Operator Theory, Academic Press.

8. Rudin, W., Functional Analysis, Second edition, Tata McGraw-Hill.
9. Sunder, V.S., Functional Analysis: Spectral Theory, Hindustan Book Agency.
10. Zhu, K., Operator Algebras, Birkhäuser.

MA5105: Algebraic Number Theory **Elective course/4 credits**

Prerequisite: Algebra III (MA4101).

Rings of algebraic integers, norms and traces, Dedekind domains and unique factorization of ideals, extensions of Dedekind domains, Galois extensions, decomposition and inertial groups, discriminants and norms of ideals, finiteness of class group, Dirichlet's unit theorem, units in quadratic fields with example, cyclotomic fields, Dedekind zeta function and analytic class number formula.

Suggested Texts:

1. Ash, R.B., A Course in Algebraic Number Theory, Dover.
2. Borevich, Z.I. and Shafarevich, I.R., Number Theory, Academic Press.
3. Frohlich, A. and Taylor, M.J., Algebraic Number Theory, Cambridge University Press.
4. Marcus, D.A., Number Fields, Springer.
5. Swinnerton-Dyer, H.P.F., A Brief Guide to Algebraic Number Theory, Cambridge University Press.

MA5106: Topics in Complex Analysis **Elective course/4 credits**

Prerequisite: Complex Analysis (MA4201).

Planar Domains: Harmonic functions, subharmonic functions, Dirichlet problem, Perron's method, Green's function, Riemann's original approach to Riemann mapping theorem.

Riemann Surfaces: Definition and examples, holomorphic function, harmonic functions, subharmonic functions, Dirichlet problem, bipolar Green's function, the uniformization theorem.

Suggested Texts:

1. Conway, J.B., Functions of One Complex Variable II, Springer.
2. Donaldson, S., Riemann Surfaces, Oxford University Press.

3. Fisher, S.D., Function Theory on Planar Domains: A Second Course in Complex Analysis, Dover.

4. Gamelin, T.W., Complex Analysis, Springer.

MA5107: Stochastic Processes **Elective course/4 credits**

Prerequisite: Probability II (MA4203).

Markov Chain: Discrete time/discrete space Markov chains - basic theory, Markov chains with stationary transition probabilities, properties of transition functions, classification of states, stationary distribution of a Markov chain, existence and uniqueness, convergence to the stationary distribution, random walks, gambler's ruin problem, transient states.

Different Processes: Poisson process, birth and death processes. finite state continuous time Markov chains, renewal processes, Poisson process as a renewal process, elementary renewal theorem, statement (without proof) of other renewal theorems, simple queueing systems, introduction to Brownian motion.

Suggested Texts:

1. Feller, W., An Introduction to Probability Theory and Its Applications, Vol. II, Wiley.

2. Hoel, P.G., Port, S.C. and Stone, C.J., Introduction to Stochastic Processes, Waveland Press, Inc.

3. Karlin, S. and Taylor, H.M., A First Course in Stochastic Processes, Academic Press.

4. Ross, S., Introduction to Probability Models, Elsevier.

MA5108: Multivariate Statistics **Elective course/4 credits**

Prerequisite: Statistical Inference (MA4107).

Distance between two random vectors, multivariate distribution function, generalized variance, properties of multivariate normal distribution and estimation of its parameters, distribution of quadratic forms, spherical and elliptical distributions, Wishart and Hotelling's T^2 distributions along with their properties, classification and discriminant analysis, multiple and partial correlation coefficients, principal component analysis, canonical correlations and canonical variables, clustering and factor analysis.

Suggested Texts:

1. Anderson, T.W., An Introduction to Multivariate Statistical Analysis, Wiley.
2. Giri, N.C., Multivariate Statistical Analysis, Academic Press.
3. Johnson, R.A. and Wichern, D.W., Applied Multivariate Statistical Analysis, Prentice-Hall of India.
4. Jolliffe, I.T., Principal Component Analysis, Springer.
5. Kshirsagar, A.M., Multivariate Analysis, Marcel Dekker.
6. Rao, C.R., Linear Statistical Inference and Its Applications, Wiley.
7. Rencher, L.C., Methods of Multivariate Analysis, Wiley.

MA5109: Nonparametric Statistics **Elective course/4 credits**

Prerequisite: Statistical Inference (MA4107).

Formulation of the problems, order statistics and their distributions, tests and confidence intervals for population quantiles, sign test, test for symmetry, signed rank test, Wilcoxon-Mann-Whitney test, Kruskal-Wallis test, run test, tests for independence, goodness-of-fit test, concepts of asymptotic efficiency, estimation of location and scale parameters.

Suggested Texts:

1. Bickel, P.J. and Doksum, K.A., Mathematical Statistics, Chapman and Hall/CRC Press.
2. Casella, G., and Berger, R.L. Statistical Inference, CRC Press.
3. Gibbons, D. and Chakraborti, S., Nonparametric Statistical Inference, CRC Press.
4. Hajek, J. and Sidak, Z., Theory of Rank Tests, Academic Press.
5. Lehmann, E.L. and D'Abrera, H.J.M., Nonparametrics: Statistical Methods Based on Ranks, Springer.

MA5110: An Introduction to Lie Algebra **Elective course/4 credits**

Prerequisite: Linear Algebra II (MA3104)

Finite dimensional Lie algebras, Ideals, homomorphisms, solvable and nilpotent Lie algebras, semisimple Lie algebras, Jordon decomposition, killing form, complete reducibility of representations, root space decomposition, root systems, Weyl group, classification theorem, construction of root systems.

Suggested Texts:

1. Humphreys, J.E. Introduction to Lie Algebras and Representation Theory, Springer-Verlag, 1972.
2. Serre, J.P. Complex Semisimple Lie Algebras, Springer, 2001.
3. Fulton, W. and Harris, J. Representation Theory: A First Course, Springer-Verlag, 1991.

MA5111: Statistical Decision Theory **Elective course/4 credits**

Prerequisite: Statistical Inference (MA4107).

Decision rules and risk, randomized and non-randomized decision rules, utility and loss functions, admissible decision rules, admissibility of Bayes' rules and generalized Bayes' rules, minimax principle, complete and essentially complete classes, invariant statistical decision problems, multiple decision problems, sequential decision problems.

Suggested Texts:

1. Berger, J.O., Statistical Decision Theory, Springer-Verlag.
2. Ferguson, T.S. and Birnbaum, Z.W., Mathematical Statistics : A Decision Theoretic Approach, Academic Press.
3. Lehmann, E.L. and Casella, G., Theory of Point Estimation, Springer.
4. Wald, A., Sequential Analysis, Dover publication.

MA5112: Theory of Sample Surveys **Elective course/4 credits**

Prerequisites: Probability I (MA2202) and Statistics I (MA3205)

Sampling frame, sampling design and sampling scheme, inclusion and exclusion probabilities, Horvitz-Thompson estimator, unequal probability sampling with and without replacement, Hansen-Hurwitz estimator, Desraj estimator, simple random sampling, systematic sampling, modified systematic sampling, inter-penetrating sub-sampling technique, stratified sampling, ratio and regression methods of estimation, cluster sampling, multi-stage sampling, two-phase sampling.

Suggested Texts:

1. Cochran, W.G., Sampling Techniques, John Wiley.
2. Mukhopadhyay, P., Theory and Methods of Survey Sampling, Prentice-Hall of India.
3. Murthy, M.N., Sampling Theory and Methods, Indian Statistical Institute.
4. Raj, D. and Chandok, P., Sample Survey Theory, Narosa Publishing House.
5. Singh, D. and Chaudhary, F.S., Sample Survey Design, New Age Publication.

MA5113: Reliability Theory **Elective course/4 credits**

Prerequisite: Consent of the Instructor.

Historical background of reliability, failure distributions, maintenance policies, hazard rate, reversed hazard rate, mean residual life, classification of distributions in terms of failure rate etc., coherent system, structure function, path sets and cut sets, structure function in terms of minimal path set and minimal cut set, equivalent structure, cold standby, warm standby and hot standby, component redundancy versus system redundancy, system signature, stochastic orders and ageing, different kinds of dependence.

Suggested Texts:

1. Barlow, R.E. and Proschan, F., Statistical Theory of Reliability and Life Testing : Probability Models, To Begin With.
2. Balagurusamy, E., Reliability Engineering, Tata McGraw-Hill.
3. Lewis, E.E., Introduction to Reliability Engineering, Wiley.
4. Lie and Xie, Stochastic Ageing and Dependence for Reliability, Springer.
5. Samaniego, F.J., System Signatures, Springer.
6. Shaked, M. and Shathikumar, J.G., Stochastic Orders, Springer.

MA5114: Riemannian Geometry **Elective course/4 credits**

Prerequisite: Differential Geometry (MA4205).

Metric: Definition of Riemannian metric and Riemannian manifolds.

Connections: Definition, Levi-Civita connection, covariant derivatives, parallel transport.

Geodesics: The concepts of geodesics, geodesics in the upper half plane, first variational formula, local existence and uniqueness of geodesics, the exponential map, Hopf-Rinow theorem.

Curvature: Curvature tensor and fundamental form, computation of curvature with examples, Ricci, sectional and scalar curvature.

Suggested Texts:

1. Do Carmo, M., Riemannian Geometry, Birkhäuser.
2. Gallot, S., Hulin, D. and Lafontaine, J., Riemannian Geometry, Springer.
3. Helgason, S., Differential Geometry, Lie Groups and Symmetric Spaces, American Mathematical Society.
4. Lee, J., Riemannian Manifolds, Springer.
5. Milnor, J.W., Morse Theory, Hindustan Book Agency.
6. Spivak M., A Comprehensive Introduction to Differential Geometry, Vols. I & II, 3rd Edition, Publish or Perish.

MA5115: Independent Study II **Elective course/4 credits**

Prerequisite: Consent of the instructor & mentor

See the end of this document for detailed guidelines.

MA5116: Introduction to PDE and Distribution Theory
Elective course/4 credits

Prerequisites: Functional Analysis (MA4102) and Ordinary Differential Equations (MA4202)

Frist-Order PDEs: Linear, semilinear, quasilinear equations; The method of characteristics; The existence and uniqueness theorem.

Distribution Theory: Test functions, distributions, basic properties of distributions, support and singular support of distributions, convolution of distributions, Fourier transform, Schwartz spaces, tempered distributions, fundamental solution.

Sobolev Spaces: Basic properties of Sobolev spaces, approximation by smooth function, extension theorem, embedding theorem, compactness theorem.

Suggested texts:

1. Pinchover, Y. and Rubinstein, J. An Introduction to Partial Differential Equations.
2. Kesavan, S. Topics in Functional Analysis and applications.
3. Evans, L. Partial Differential Equations.
4. Brezis, H. Functional Analysis, Sobolev Spaces and Partial Differential Equations.

MA5117: Introduction to Ergodic Theory
Elective course/4 credits

Prerequisite: Analysis IV (MA3204) and Functional analysis (MA4102).

Ergodicity, recurrence and mixing: Measure preserving transformation, Recurrence, Ergod- icity, The Mean Ergodicity Theorem, Pointwise ergodic theorem, Strong & weak mixing.

Invariant measures for continuous maps: Existence of invariant measures, Ergodic decom- position, Unique ergodicity, Measure rigidity & equidistribution.

Further topics (if time permits): Conditional measures, Factors & joining.

Suggested Texts:

1. Walters, P. An Introduction to ergodic theory, Springer.
2. Einsiedler, M and Ward, T. Ergodic theory with a view toward number theory, Springer.

MA5118: Commutative Algebra
Elective course/4 credits

Prerequisite: Algebra II (MA3203).

Direct sum and tensor product of modules, finitely generated modules, Nakayama lemma, exact sequence of modules.

Projective, injective, free and flat modules. Direct limit and inverse limit of modules, Tor and Ext groups for modules.

Graded rings and modules, homogeneous prime and maximal ideals. Rings and modules of fractions, local ring, homogeneous localization of graded rings and modules.

Modules of finite lengths, Noetherian and Artinian rings and modules, Hilbert's basis theorem, finitely presented modules, locally free modules.

Primary decomposition, support of a module, minimal and associated prime ideals. Height of a prime ideal, Krull dimension of a ring.

Integral extension, going-up and going-down theorems, integral closure, normal domain, discrete valuation rings. Noethers normalization lemma, Hilberts Nullstellensatz.

Linear topology and ideal-adic completions. Derivations and differentials, module of Kahler differentials. Regular local ring, differential criterion for regularity, Jacobian criterion for regularity.

Suggested Texts:

1. Atiyah, M.F. and MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley.
2. Gopalakrishnan, N.S., Commutative Algebra, Oxonian Press.
3. Singh, Balwant. Basic Commutative Algebra, World Scientific.
4. Matsumura, H., Commutative Ring Theory, Cambridge University Press.
5. Matsumura, H., Commutative Algebra, Benjamin/Cummings Publishing Co., Inc., Reading, Mass., 1980.

10th Semester

MA5201: MS Project II **Core course/12 credits**

See the end of the document for detailed guidelines.

MA5202 : Algebraic Geometry **Elective course/4 credits**

Prerequisite: Algebra III (MA4101).

The projective plane, Bezout's theorem, application (addition law on cubic curves) and proof, affine varieties, quasi-projective varieties, images of projective varieties, dimension of varieties, nonsingular projective curves, divisors, Picard group, hyper-elliptic curves, differentials, the canonical divisor, genus of a curve, Riemann-Roch theorem, Riemann-Hurwitz formula.

Suggested Texts:

1. Fulton, W., Algebraic Curves : An Introduction to Algebraic Geometry, Addison-Wesley.
2. Harris, J., Algebraic Geometry : A First Course, Springer-Verlag.
3. Kendig, K., Elementary Algebraic Geometry, Springer-Verlag.
4. Musili, C., Algebraic Geometry for Beginners, Hindustan Book Agency.
5. Shafarevich, I.R., Basic Algebraic Geometry, Springer.

MA5203: Topics in Operator Theory **Elective course/4 credits**

Prerequisite: Operator Theory (MA5104).

Topics to be chosen from the following texts:

1. Agler, J. and McCarthy, J.E., Pick Interpolation and Hilbert Function Spaces, American Mathematical Society.
2. Conway, J.B., Subnormal Operators, Mathematical Surveys and Monographs, American Mathematical Society.
3. Paulsen, V., Completely Bounded Maps and Operator Algebras, Cambridge University Press.
4. Sz-Nagy, B., Foias, C., Bercovici, H. and Kerchy, L., Harmonic Analysis of Operators on Hilbert Space, Springer.

MA5204 : Several Complex Variables **Elective course/4 credits**

Prerequisites: Complex Analysis (MA4201) and Functional Analysis (MA4102).

Holomorphic functions, separately holomorphic functions, Hartog's theorem, homomorphically convex domains and its characterisations, inhomogeneous Cauchy-Riemann equation, Hartog's phenomenon, domains of holomorphy, plurisubharmonic functions, pseudoconvex domains and its characterisations, Levi problem, solution of $\bar{\partial}$-problem by Hörmander's method.

Suggested Texts:

1. Hormander, L., An Introduction to Complex Analysis in Several Variables, Elsevier.
2. Krantz, S., Function Theory of Several Complex Variables, AMS Chelsea Publishing.
3. Ohsawa, T., Analysis of Several Complex variables, Mathematical Monographs, American Mathematical Society.

MA5205: Advanced Partial Differential Equations **Elective course/4 credits**

Prerequisite: Partial Differential Equations (MA5102).

Sobolev Spaces: Distribution theory, Sobolev spaces, embedding theorems, Rellich's lemma, trace theorem.

Elliptic Equations: Second order elliptic equations, weak formulation, Lax-Milgram lemma, existence.

Hamilton-Jacobi Equations: Hopf-Lax formula, weak solution of Hamilton-Jacobi equation and its uniqueness.

Conservation Laws: Weak solutions, Rankine-Hugoniot condition, shocks, Lax-Oleinik formula, entropy condition and uniqueness of entropy solution.

Suggested Texts:

1. Attouch. H., Buttazzo. G. and Michaille. G., Variational Analysis in Sobolev and BV Spaces: Applications to PDEs and Optimization, SIAM.
2. Brezis. H., Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.
3. Evans. C.L., Partial Differential Equations, American Mathematical Society.
4. Kesavan. S., Topics in Functional Analysis and Applications, Wiley.
5. Leoni. G., A First Course in Sobolev Spaces, American Mathematical Society.

MA5206: Topics in Analysis **Elective course/4 credits**

Prerequisite: Consent of the Instructor.

Relevant papers will be given by the Instructor.

MA5207: Topology and Geometry **Elective course/4 credits**

Prerequisite: Consent of the Instructor.

Algebraic Topology: Poincaré Duality, Kunneth product formula, universal coefficient theorem.

Differential Topology: Transversality and Morse-Sard theorem, distributions and integrability, Frobenius theorem.

Euler Class: Orientation, intersection number, Euler characteristic, Lefschetz fixed point theorem, index of vector field, Poincaré-Hopf index theorem, Euler form and Euler class, Euler class of oriented manifolds via Poincaré duality.

Suggested Texts:

1. Hatcher, A.W., Algebraic Topology, Cambridge University Press.
2. Milnor, J.W. and Stasheff, J., Characteristic Classes, HBA
3. Guillemin, V. and Pollack, A., Differential Topology, Prentice-Hall
4. Hirsch. M., Differential Topology, Springer
5. Bott, R. and Tu, L.W., Differential Forms and Algebraic Topology, Springer

MA5208: Introduction to Bayesian Analysis **Elective course/4 credits**

Prerequisite: Statistical Inference (MA4107).

Statistical models and prior information, inference based on the posterior distribution, choice of the prior distribution, point and interval estimation, hypothesis testing and the Bayes' factor, model selection, MCMC, hierarchical models and exchangeability.

Suggested Texts:

1. Albert, J., Bayesian Computation with R, Springer.
2. Robert, C. and Casella, G., Monte Carlo Statistical Methods, Springer.
3. Ghosh, J.K., Delampady, M. and Sengupta, T., An Introduction to Bayesian Analysis, Springer.
4. Ho, P.D., A First Course in Bayesian Statistical Methods, Springer.
5. Lee, P., Bayesian Statistics: An Introduction, Oxford University Press.

MA5209: Time Series Analysis **Elective course/4 credits**

Prerequisites: Statistical Inference (MA4107) and Functional Analysis (MA4102).

Introduction: Review of various components of time series, plots and descriptive statistics, discrete-parameter stochastic processes– strong and weak stationarity, autocovariance and autocorrelation.

Spectral Analysis and Different Processes: Spectral analysis of weakly stationary processes– periodogram, fast Fourier transform; Moving average, autoregressive, autoregressive moving average (ARMA) and autoregressive integrated moving average processes (ARIMA); Box-Jenkins model, state-space model.

Forecasting and Model Selection: Linear filters, signal processing through filters, inference in ARMA and ARIMA models; Forecasting– ARIMA and state-space models, Kalman filter; Model building– residuals and diagnostic checking; Model selection– strategies for missing data.

Time-frequency Analysis: Short-term Fourier transform, wavelets, data analysis with computer packages.

Suggested Texts:

1. Brockwell, P.J. and Davis, R.A., Introduction to Time Series and Forecasting, Springer.

2. Fuller, W.A., Introduction to Statistical Time Series, Wiley-Blackwell.

3. Shumway, R.H. and Stoffer, D.S., Time Series Analysis and Its Applications, Springer.

MA5210: Advanced Data Structures and Algorithms
Elective course/4 credits

Prerequisite: Programming and Data Structures (CS4101).

Introduction and Basic Concepts: complexity measures, worst-case and average-case complexity functions, quick review of basic data structures.

Advanced Trees: AVL tree, KD Trees, B-Trees.

Lists: Link list, priority queues/heaps, hashing, dictionaries.

Graphs: Depth-first and breadth-first search, shortest path algorithms, minimal spanning tree algorithms.

Sorting and Selection: Insertion sort, selection sort, bubble sort, quick sort, merge sort, radix sort, lower bound for sorting.

Algorithm Paradigms: Greedy, divide and conquer, dynamic programming.

Suggested Texts:

1. Aho, A.V., Hopcraft J.E., and Ullman, J.D., Design and Analysis of Algorithms, Addison-Wesley.

2. Cormen, T.H., Leiserson, C.E., Rivest, R.L. and Stein C., Introduction to Algorithms, Prentice-Hall.

3. Kleinberg, J and Tardos, É., Algorithm Design, Addison-Wesley.

MA5211: High Dimensional Statistics **Elective course/4 credits**

Prerequisite: Statistical Inference (MA4107)

Curse of dimensionality with examples, motivating examples from genetics, machine learning and other fields. Regression techniques: Subset selection methods, shrinkage methods, LASSO and its generalizations with computational techniques and some asymptotics (if time permits), LARS. Hypothesis testing: Multiple testing, Bonferroni correction, FWER, FDR and their properties. Sparse methods: sparse PCA and sparse LDA with emphasis on the high dimensional low sample size (HDLSS) situation.

Suggested Texts:

1. T. Hastie, R. Tibshirani and J. Friedman, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition, Springer Series in Statistics.
2. G. James, D. Witten, T. Hastie and R. Tibshirani, An Introduction to Statistical Learning: with Applications in R, Springer Texts in Statistics.
3. C. Giraud, Introduction to High Dimensional Statistics, Chapman and Hall.
4. T. Hastie, R. Tibshirani and M. Wainwright, Statistical Learning with Sparsity: The Lasso and Generalizations, Chapman and Hall.
5. S. Dudoit and M. J. van der Laan, Multiple Testing Procedures with Applications to Genomics, Springer Series in Statistics.
6. P. H. Westfall and S. S. Young, Resampling-Based Multiple Testing: Examples and Methods for p-Value Adjustment, Wiley.

MA5212: Regression Analysis **Elective course/4 credits**

Prerequisite: Statistical Inference (MA4107).

Classical linear regression model, estimation and confidence interval of parameters, Gauss-Markov theorem, estimable parametric function and its BLUE, least square estimation with restrictions on parameters, testing of regression estimators, heteroscedasticity, variance stabilizing transformation, method of detecting outlier, Box-Cox method, multicollinearity and Ridge regression, autocorrelation and Durbin-Watson test, Cochrane-Orcutt method, indicator variables, non-linear regression, logistic regression.

Suggested Texts:

1. Brockwell, P. J. and Davis, R. A., Introduction to Time Series and Forecasting, Second Edition, Springer.

2. Drapper, N.R. and Smith,H., Applied Regression Analysis, John Wiley.

3. Gujarati,.N. and Porter, D.C., Basic Econometrics, McGraw-Hill.

4. Montgomery, D.C., Peck, E.A. and Vining, G.G., Introduction to Linear Regression Analysis, Wiley.

5. Rao, C.R., Linear Statistical Inference and Its Applications, Wiley.

6. Sengupta, D. and Jammalamadaka, S.R., Linear Models : An Integrated Approach, World Scientific.

MA5213: Sobolev Spaces: Theory and Applications
Elective course/4 credits

Prerequisites: Analysis V (MA4103), Functional Analysis (MA4102); Knowledge of Partial Differential Equations, though not necessary, would be useful.

Sobolev Spaces: Weak derivative, Sobolev spaces, approximation by smooth functions, approximations up to the boundary, extension of Sobolev functions, embedding theorems, compact embeddings, Rellich Theorem, Poincare inequalities, traces of Sobolev functions.

Elliptic PDE (L^2 theory): Elliptic equations, Lax-Milgram Theorem, existence of weak solutions, regularity of solutions, maximum principles, eigenvalue problem.

Suggested Texts:

1. Adams, R. A. and Fournier, J. J. F., Sobolev Spaces. Second edition. Pure and Applied Mathematics (Amsterdam), 140, Elsevier/Academic Press, Amsterdam, 2003.

2. Evans, L. C., Partial Differential Equations. Second Edition, AMS, 2010.

3. Evans, L. C. and Gariepy, R. F., Measure Theory and Fine Properties of Functions. CRC Press, 1992.

4. Gilbarg, D. and Trudinger, N., Elliptic Partial Differential Equations of Second Order. Second Edition, Springer, 1983.

5. Kesavan, S., Topics in Functional Analysis and Applications. Wiley Eastern Limited, 1989.

6. Leoni, G., A First Course in Sobolev Spaces. AMS, 2009

MA5214: Principal Bundles & Representation Ring
Elective course/4 credits

Prerequisite: Consent of the Instructor.

Part I: Principal Bundles

G-spaces, orbit category, principal G-bundles, properties, Vector bundles, properties, associated bundle constructions, classifying spaces, construction and existence, May's bar construction, Elmendorf's theorem.

Part II: Representation Ring

Review of (compact) Lie groups and Cartan subgroups, Weyl group, Riemannian geometry of Lie groups, Cartan's theorem, representation ring of (compact) Lie groups, augmentation ideal.

Suggested Texts:

1. Bott, R. Homogeneous Vector Bundles; Annals of Mathematics, 2nd Ser., Vol. 66, No. 2 (1957).

2. Elmendorf, A. Systems of Fixed Point Sets; Transactions of the American Mathematical Society, Vol. 277, No. 1 (1983).

3. Hsiang, W.Y. Lectures on Lie Groups; Singapore: World Scientific (2000).

4. Kirillov, A. Jr. An Introduction to Lie Groups and Lie Algebras; Cambridge Studies in Advanced Mathematics, Vol. 113 (2008).

5. May et al. Equivariant Homotopy and Cohomology Theory; CBMS Vol. 91, American Mathematical Society (1996).

6. Milnor, J. Construction of Universal Bundles I; Annals of Mathematics, Vol. 63, No. 2 (1956).

7. Milnor, J. Construction of Universal Bundles II; Annals of Mathematics, Vol. 63, No. 3 (1956).

8. Milnor, J. Curvatures of Left Invariant Metrics on Lie Groups; Advances in Mathematics, Vol. 21 (1976).

9. Husemoller et al. Basic Bundle Theory and K-Cohomology Invariants, Lecture Notes in Physics book series (LNP, Vol. 726) Springer (2008).

10. Segal, G. The Representation-ring of a Compact Lie Group; Publ. IHES, tome 34 (1968).

11. Steenrod, N. The Topology of Fibre Bundles; Princeton University Press (1999).

MA5215: Analytic Number Theory **Elective course/4 credits**

Prerequisite: Algebra III (MA4101), Representation Theory of Finite Groups (MA4204), Topics in Complex Analysis (MA5106)

Divergence of the sum of the reciprocals of the primes, Chebyshev's estimates for the prime counting function, Bertrand's postulate.

Dirichlet convolution, Möbius inversion formula, the ring of arithmetic functions, Dirichlet's hyperbola method, analytic properties of Dirichlet series, Mertens' three estimates, properties of the Riemann Zeta function, Dirichlet's theorem on primes in arithmetic progression, the Prime Number Theorem.

Elements of Sieve Theory, the sieve of Eratostenes, the Brun sieve, convergence of the sum of the reciprocals of the twin primes.

Modular groups, classical modular functions and modular forms, the fundamental domain of the full modular group, the valence formula, the dimension of the space of modular forms, cusp forms, Hecke operators.

Suggested textbooks:

1. Luca F., De Koninck J-M., Analytic Number Theory
2. Iwaniec H., Kowalski E., Analytic Number Theory
3. Bruinier J-H., van der Geer G., Harder G., Zagier D., The 1-2-3 of Modular Forms

Suggested References:

1. Davenport H., Multiplicative Number Theory
2. Serre J-P., A Course in Arithmetic
3. Ireland K., Rosen M., A Classical Introduction to Modern Number Theory

4. Tenenbaum G., Introduction to Analytic and Probabilistic Number Theory
5. Koukoulopoulos D., The Distribution of Prime Numbers
6. Friedlander J., Iwaniec H., Opera de Cribro
7. Apostol T., Introduction to Analytic Number Theory

MA5216: Independent Study III **Elective course/4 credits**

Prerequisite: Consent of the instructor & mentor

See the end of this document for detailed guidelines.

MA5217: Independent Study IV **Elective course/4 credits**

Prerequisite: Consent of the instructor & mentor

See the end of this document for detailed guidelines.

MS Projects (MA5101, MA5201) **Core course/8 & 12 credits**

This course is available to 5th year BS-MS students as a core course. The guidelines are as follows:

1. *Choosing a supervisor*

   a. A student wishing to do an MS project should talk to faculty members in DMS and decide on whom to work with. This should have the approval of the chosen supervisor.

   b. If you choose to work with someone from outside DMS or outside the institute, then you must have a co-supervisor from within DMS, without whom this project will not be officially acknowledged by the department.

   c. The choice of the supervisor (and co-supervisor if applicable) should be conveyed to the course co-ordinator(s) (as displayed on welearn) within the **first seven days** of the commencement of the semester. This should be done via an e-mail to the course co-ordinator(s) with a copy to the supervisor(s).

2. *Contact hours & responsibility*

   a. It is an **8 credit** course (for 9th semester) and a **12 credit** course (for 10th semester). The responsibility is on the students to meet the supervisor for at least **2 contact hours** per week. The meeting mode and place may be mutually decided by the supervisor and the student.

b. Failure to meet the instructor for at least **three weeks** may result in failure for the project.

c. In both the semesters, the student needs to submit a draft of the MS project at least **seven days** before the MS presentation & viva. Lack of submission of a draft will result in failure for the project. A draft deemed unsuitable by the supervisor(s) would have to be submitted again before the presentation.

3. *Grading*

a. If the supervisor is from DMS, then the DMS supervisor will be responsible for timely submission of grades to the course co-ordinators.

b. If the supervisor is from outside DMS, then the DMS co-supervisor will be responsible for timely submission of grades to the course co-ordinators.

c. The final grades will be arrived at based on a final presentation and viva, where apart from the supervisor(s) another faculty expert will also be present. The final presentation committee shall also take into consideration the cumulative effort of the student, level of understanding of the topic before arriving at the final grade.

Independent Study Courses (MA4208, MA5115, MA5216, MA5217) Elective course/4 credits

This course is available to 5th year BS-MS students across departments. It is also currently available to 4th year DMS BS-MS students as an elective. The guidelines are as follows:

1. *Choosing a mentor (for DMS 5th year students)*

a. A student may choose to do an independent study course with a faculty mentor from IISER Kolkata. If the MS project supervisor is from DMS, then this mentor should be chosen in consultation and agreement with the MS project supervisor. If the MS project supervisor is from outside DMS, then the choice of mentor should be made in consultation with the DMS co-supervisor (refer to point 1 in the MS project guidelines).

b. The choice of the mentor should be conveyed to the course co-ordinator(s) (as displayed on welearn) within the **first seven days** of the commencement of the semester. This should be done via an e-mail to the course co-ordinator(s) with a copy to the mentor.

2. *Choosing a mentor (for non-DMS students and 4th year DMS BS-MS students)*

   a. A non-DMS student or a 4th year DMS BS-MS student may do an independent study course under a DMS faculty mentor.

   b. The choice of the mentor should be conveyed to the course co-ordinator(s) (as displayed on welearn) within the **first seven days** of the commencement of the semester.

3. *Contact hours & responsibility*

   a. It is a **4 credit** course. The responsibility is on the students to meet the mentor for at least **2 contact hours** per week. The meeting mode and place may be mutually decided by the instructor and the student.

   b. Failure to meet the mentor for at least **three weeks** may result in failure for the course.

4. *Syllabus & Grading*

   a. As this is a course, a syllabus has to be decided by the mentor and notified to the course co-ordinator(s) (as displayed on welearn) within the **first seven days** of the commencement of the semester. Only after the approval of this syllabus by DMS UGAC, in consultation of course co-ordinator(s), should the student be allowed to register for this course.

   b. There will be **two** written examinations: mid-sem and final. The entire grade for the course will be based on these two exams.