List of Courses
2 year M.Sc Curriculum

Department of Mathematics and Statistics
Indian Institute of Science Education and Research Kolkata

To be implemented from 21MSc batch of students.

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The Aim of This Syllabus

This document describes the courses in the proposed two-year M. S. program by the Department of Mathematics and Statistics (DMS) at Indian Institute of Science Education and Research (IISER) Kolkata. The curiosity of understanding fundamental mathematical phenomena has always been at the core of knowledge seekers. In the present era, due to various applications of mathematics in biology, computer science, linguistics, apart from its classical use in physics and other basic sciences, a solid understanding of basic mathematics has become more important. In particular, the courses in our proposed M. S. program will provide fundamental knowledge in mathematics beyond the bachelors level with rigorous training.

A few salient features of the curriculum are the following:

- *Core Courses:* These consist of Analysis (Real Analysis, Complex Analysis, Functional Analysis), Algebra, Geometry. It is desirable that a student in Mathematics masters knows these topics well.

- *Electives:* The Department offers a diverse bouquet of elective courses that range from certain introductory course in advanced topics to the frontiers of current research. Most of the elective courses are designed keeping research towards that as focus. Therefore, the students will get certain views toward current research in the elective courses, which may play certain role in their own viewpoint of research.

- *Masters Project:* In the fourth semester a student of this two-year M. S. program will get a hands-on experience of research as a masters' project under a faculty member of the DMS.

- All 4-credit theory courses will have 3 hours of theory and 1 hour of tutorial.

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DMS Course Structure for M.Sc (2 years) Program
IISER Kolkata

1st Semester: 20 credits

- MA3101: Analysis III (4 credits)
- MA3102: Algebra I (4 credits)
- MA3106: Topology and Metric spaces (4 credits)
- MA3104: Linear Algebra II (4 credits)
- MA3108: Stochastic Processes and Its Applications (4 credits)

2nd Semester: 20 credits

- MA3202: Algebra II (4 credits)
- MA3203: Analysis IV (4 credits)
- MA3205: Geometry of Curves and Surfaces (4 credits)
- MA4201: Complex Analysis (4 credits)
- MA4202: Ordinary Differential Equations (4 credits)

3rd Semester: 20 credits

- MA4101: Algebra III (4 credits)
- MA4102: Functional Analysis (4 credits)
- MA4103: Analysis V (4 credits)
- MA4104: Algebraic Topology (4 credits)
- MA5102: Partial Differential Equations (4 credits)

4th Semester: 20 credits

- MA4203: Probability Theory (4 credits)
- MA4205: Differential Geometry (4 credits)
- MAxxxx: Departmental Elective I (4 credits)
- MAxxxx: Departmental Elective II (4 credits)
- MA4210: MS Project (4 credits)

Group of Electives

Representation Theory of Finite Groups (MA4207)
Independent Study I (MA4208)
Algebraic Geometry (MA5202)
Topics in Operator Theory (MA5203)
Several Complex Variables (MA5204)
Advanced Partial Differential Equations (MA5205)
Topics in Analysis (MA5206)
Topology and Geometry (MA5207)
Sobolev Spaces: Theory and Applications (MA5213)
Commutative Algebra (MA5214)

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M.Sc Courses - Syllabus

1st Semester

MA3101: Analysis III **Core course/4 credits**

This is a course on calculus of several variables.
Topology in $\mathbb{R}^n$: Open sets, closed sets, compact sets, Heine-Borel theorem, path connectedness in $\mathbb{R}^n$.

Differential Calculus: Directional derivatives and its drawbacks, total derivative, comparison with differentiability on $\mathbb{R}$, chain rule and its applications, C^k functions, mixed derivatives, Taylor's theorem, smooth functions with compact supports, inverse function theorem, implicit function theorem and the rank theorem, examples, maxima and minima, critical point of the Hessian, constrained extrema and Lagrange's multipliers, examples.

Integral Calculus: Line integrals, behaviour of line integral under a change of parameter, independence of path, conditions for a vector field to be a gradient, concept of potential and its construction on convex sets, multiple Riemann integrals, Fubini's theorem (statement only), change of variables (statement only).

Suggested Texts:

1. Apostol, T.M.: Calculus II, Wiley India Pvt. Ltd.
2. Spivak, M.: Calculus on Manifolds, Westview Press
3. Rudin, W.: Principles of Mathematical Analysis, McGraw-Hill

MA3102: Algebra I **Core course/4 credits**

This is a course on group theory.
Groups: Definition of groups, subgroups, group homomorphisms and isomorphisms, normal subgroups, quotient groups, Lagrange's theorem, isomorphism theorems, direct sum of abelian groups, direct products, group as symmetries, free group (if time permits).

Group Action: Group actions, conjugacy classes, orbits and stabilizers, class equations.

Symmetric Groups: Symmetric groups, simple groups, simplicity of A_n with $n > 4$.

Sylow's theorems: Three theorems of Sylow, classification of finitely generated abelian groups.

Suggested Texts:

1. Artin, M., Algebra, Prentice-Hall.
2. Dummit, D.S. and Foote, R.M., Abstract Algebra, Wiley.
3. Fraleigh, J.B., A First Course in Abstract Algebra, Narosa Publishers.
4. Gopalakrishnan, N.S., University Algebra, New Age International.
5. Herstein, I.N., Topics in Algebra, Wiley.
6. Hungerford, T.W., Algebra, Springer-Verlag.
7. Malik, D.S., Mordeson, J.M. and Sen, M.K., Fundamentals of Abstract Algebra, McGraw-Hill.

MA3104: Linear Algebra II **Core course/4 credits**

This is the sequel to a course on basic linear algebra (refer to MA2102 from BS-MS syllabus).
Eigenvalues: Annihilating polynomials, minimal polynomials, invariant subspaces, simultaneous diagonalization and triangularization, direct sum decompositions, primary decomposition theorem.

Canonical Forms: Cyclic decompositions and the rational form, Jordan canonical form, computation of invariant factors.

Matrix Decomposition: Orthogonal, unitary and Hermitian matrices, unitary similarity, Schur's triangularization theorem, spectral theorem for normal matrices, positive definite matrices, polar decomposition, singular value decomposition.

Bilinearity: Bilinear maps, tensors and tensor product with emphasize on inner product space.

Exterior Forms: Quadratic forms, bilinear forms, symmetric and alternating forms, determinant function and uniqueness of determinant.

Suggested Texts:

1. Axler, S., Linear Algebra Done Right, Springer-Verlag.
2. Friedberg, S.H., Insel, A.J. and Spence, L.E., Linear Algebra, Prentice-Hall.
3. Horn, R. and Johnson, C.R., Matrix Analysis, Cambridge University Press.
4. Hoffman, K. and Kunze, R., Linear Algebra (2nd Edition), Prentice-Hall.
5. Lang, S., Introduction to Linear Algebra (2nd Edition), Springer-Verlag.
6. Lax, P.D., Linear Algebra and Its Applications, Wiley.

7. Rao, A.R. and Bhimasankaran, P., Linear Algebra (2nd Edition), Hindustan Book Agency.

8. Roman, S., Advanced Linear Algebra, Springer.

9. Shafarevich, I.R. and Remizov, A.O., Linear Algebra and Geometry, Springer.

MA3106: Topology & Metric Spaces **Core course/4 credits**

Metric Spaces: Metric space topology, equivalent metrics, sequences, complete metric spaces, limits and continuity, uniform continuity, extension of uniformly continuous functions.

Topological Spaces: Definition, examples, bases, sub-bases, product topology, subspace topology, metric topology, quotient topology, second countability and separability.

Continuity: Continuous functions on topological spaces, homeomorphisms.

Connectedness: Definition, example, path connectedness and local connectedness.

Compactness: Definition, limit point compactness, sequential compactness, net and directed set, local compactness, Tychonoff theorem, Stone-Weierstrass theorem, Arzela-Ascoli theorem.

Separation Axioms: Hausdorff, regular and normal spaces; Urysohn lemma and Tietze extension theorem; compactification.

Metrizability: Urysohn metrization theorem.

Suggested Texts:

1. Armstrong, M.A., Basic Topology, Springer-Verlag.

2. Dugundji, J., Topology, Allyn and Bacon Series in Advanced Mathematics, Allyn & Bacon.

3. Kelley, J.L., General Topology, Springer-Verlag.

4. Munkres, J.R., Topology (2nd Edition), Prentice-Hall.

5. Simmons, G.F., Introduction to Topology and Modern Analysis, Tata McGraw-Hill.

MA3108: Stochastic Processes and Its Applications **Core course/4 credits**

Quick review of probability theory: Probability spaces, random variables, probability distributions, expectations, transforms and generating functions, convergence, LLNs, CLT.

Introduction to stochastic processes: Definition and examples; classification of random processes.

Discrete-time Markov chain: Definition and examples; homogeneous Markov chain; transition probability matrix, Chapman-Kolmogorov equations; limiting probabilities; ergodicity; stationary distribution; random walk; gambler?s ruin problem; different applications of Markov chains including an introduction to MCMC techniques.

Discrete-time Martingales: Definitions and examples; sub/super-martingales; stopping times; optional sampling theorem; martingale inequalities; reverse martingales; convergence theorems; applications.

Continuous-time Markov chain: Definition and examples; Birth & death process; Poisson process; Holding times and transitions; Transition rate and transition probabilities; Kolmogorov forward and backward equations; infinitesimal generator and jump chain; classification of states; (if time permits) long-run behaviour of continuous-time Markov chains.

Suggested Texts:

1. Ross; Introduction to Probability Models, Academic Press.
2. Taylor and Karlin; An Introduction to Stochastic Modelling, Academic Press.
3. Ross; Stochastic Processes, Wiley India.
4. Karlin and Taylor; A First Course in Stochastic Processes, Academic Press.
5. Hoel, Port, and Stone; Introduction to Stochastic Processes, Waveland Press.
6. Resnick; Adventures in stochastic processes, Birkhauser.
7. Billingsley; Probability and Measure, Wiley.

2nd Semester

MA3202: Algebra II **Core course/4 credits**

This is a course on rings and modules.

Rings and Ideals: Rings and ring homomorphism, ideals, quotient rings, zero-divisors, units, prime and maximal ideals, nilradical and Jacobson radical operations on ideals, extension and contraction, division in domains, g.c.d. and l.c.m., division algorithm, Euclidean domain, unique factorization domain, principal ideal domain.

Modules: Modules and module homomorphisms, submodule and quotient modules, operations on submodules, direct sum and product, finitely generated modules, classification of finitely generated modules over PIDs, exact sequences of modules, tensor product of modules, canonical forms.

Suggested Texts:

1. Artin, M., Algebra, Prentice-Hall.
2. Atiyah, M.F. and MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley.
3. Dummit, D.S. and Foote, R.M., Abstract Algebra, Wiley.
4. Eisenbud, D., Commutative Algebra with a view towards Algebraic Geometry, Springer-Verlag.
5. Gopalakrishnan, N.S., Commutative Algebra, Oxonian Press.
6. Kunz, E., Introduction to Commutative Algebra and Algebraic Geometry, Birkhäuser.
7. Luthar, I.S. and Passi, I.B.S., Algebra, Vol. 2: Rings, Narosa Publishing House.
8. Luthar, I.S. and Passi, I.B.S., Algebra, Vol. 3: Modules, Narosa Publishing House.
9. Matsumura, H., Commutative Ring Theory, Cambridge University Press.
10. Reid, M., Undergraduate Commutative Algebra, London Mathematical Society Student Texts (29), Cambridge University Press.
11. Sharp, R.Y., Steps in Commutative Algebra, London Mathematical Society Student Texts (19), Cambridge University Press.

MA3203: Analysis IV **Core course/4 credits**

This is a course on measures and integrals.
Introduction: Drawbacks of Riemann integration, measurement of length– introductory remarks.

Abstract Measures: Algebra, σ-algebra and Borel σ-algebra, outer measure, measure, Caratheodory extension Theorem and construction of Lebesgue measure on $\mathbb{R}^n$ as an application, measure space, measurable set and measurable function.

Integration Theory: Definition and properties of Lebesgue integral, basic convergence theorems– monotone convergence theorem, Fatou's lemma and dominated convergence theorem.

Borel Measure: Regularity properties of Borel measure, Radon measure, Caratheodory's criterion; Continuity properties of measurable functions– Lusin's and Egoroff's theorems.

L^p Spaces: Fundamental inequalities - Hölder's inequality, Jensen's inequality and Minkowski's inequality, definition of L^p spaces, completeness, approximation by continuous functions.

Product Measure: Measurability in product spaces, product measures, Fubini and Fubini-Tonelli theorems, polar coordinates and change of variable theorem.

Suggested Texts:

1. De Barra, G., Measure Theory and Integration, New Age International Publishers.
2. Evans, L.C. and Gariepy, R.F., Measure Theory and Fine Properties of Functions, CRC Press.
3. Folland, G.B., Real Analysis: Modern Techniques and Their Applications (2nd Edition), Wiley-Interscience.
4. Kantorovitz, S., Introduction to Modern Analysis, Oxford University Press.
5. Rana, I.K., An Introduction to Measure and Integration, Narosa Publishers.
6. Royden, H.L., Real Analysis, Prentice-Hall.
7. Rudin, W., Real and Complex Analysis, McGraw-Hill.

MA3205: Geometry of Curves and Surfaces **Core course/4 Credits**

Part I : Curves

Curves: Parametrized and regular curves, arc length, parametrization by arc length.

Local Theory: Tangent-normal-binormal frame, curvature, torsion, fundamental theorems of local theory of plane and space curves.

Global Theory: Simple curves, Jordan curve theorem (without proof), isoperimetric inequality, four-vertex theorem.

Part II : Surfaces

Surfaces: Parametrization, change of parameters, smooth functions, tangent plane, differential, diffeomorphism, inverse and implicit function theorems.

Second Fundamental Form and Curvature: Gauss map; oriented surfaces; second fundamental form; Gauss, mean and principal curvatures; normal sections.

Integration on Surface: Definition of integral, partitions of unity, change of variables formula, divergence theorem.

Geometry of Surfaces: Rigid motions and isometries, Gauss's Theorema Egregium, geodesics.

Gauss-Bonnet Theorem : Index of a vector field at an isolated zero, Euler characteristic (if time permits).

Suggested Texts:

1. Berger, M. and Gostiaux, B., Differential Geometry: Manifolds, Curves and Surfaces, Springer-Verlag.
2. Do Carmo, M.P., Differential Geometry of Curves and Surfaces, Prentice-Hall.
3. Montiel, S. and Ros, A., Curves and Surfaces, Graduate Studies in Mathematics, Vol. 69, American Mathematical Society.
4. O'Neill, B., Elementary Differential Geometry (2nd Edition), Academic Press.
5. Pressley, A., Elementary Differential Geometry, Springer-Verlag.
6. Thorpe, J. A., Elementary Topics in Differential Geometry, Springer-Verlag.

MA4201: Complex Analysis **Core course/4 credits**

Complex Number System: Field of complex numbers, polar representations, power, roots, complex exponential, complex logarithm, extended complex plane, Riemann sphere and stereographic projection.

Analytic Functions: Definitions, Cauchy-Riemann equations, harmonic functions.

Complex Integration: Riemann-Stieltjes integration, power series representation of analytic functions, zeros of an analytic function, winding number, Cauchy's integral formula, Cauchy estimates and Liouville theorem, Cauchy's theorem, Morera's theorem, open mapping theorem, maximum modulus theorem, Schwarz's lemma.

Singularities: Classification of singularities, Laurent series, Casorati-Weierstrass theorem, residues, evaluation of definite integrals using residue theorem, meromorphic functions, argument principle, Rouché's theorem.

Conformal Mappings: Definitions, conformal maps and geometry of Möbius transformations, normality and compactness, Riemann mapping theorem.

Gamma Functions: (If time permits) Gamma functions through functional equations, elementary properties.

Suggested Texts:

1. Ahlfors, L.V., Complex Analysis, McGraw-Hill.
2. Conway, J.B., Functions of One Complex Variable, Springer-Verlag.
3. Gamelin, T.W., Complex Analysis, Springer.
4. Greene, R.E. and Krantz, S.G., Function Theory of One Complex Variable, American Mathematical Society.
5. Lang, S., Complex Analysis, Springer-Verlag.
6. Marsden, J.E., Basic Complex Analysis, W.H. Freeman & Co.
7. Narasimhan, R., Complex Analysis in One Variable, Birkhäuser-Verlag.
8. Rao, M., Stetkae, H. and Fournais, S., Complex Analysis : An Invitation, World Scientific.
9. Rudin, W., Real and Complex Analysis, McGraw-Hill.
10. Stein, E. M. and Shakarchi, R., Complex Analysis, Princeton Lectures in Analysis.

MA4202: Ordinary Differential Equations **Core course/4 credits**

Fundamental Theory: Existence of solutions under continuity, existence and uniqueness under Lipschitz condition, non-uniqueness and Kneser's theorem, extension of solutions, dependence of solutions with respect to initial data and parameter, flow of an ordinary differential equation.

Boundary-Value Problems of Linear Differential Equations of the Second Order: Zeros of solutions, Sturm-Liouville problems, eigenvalue problems, eigenfunction expansions.

Linear System: Exponentials of operators, fundamental theorem for linear systems, linear systems in $\mathbb{R}^2$.

Stability Theory: Stable, unstable and asymptotically stable points; Liapunov functions; Stable manifolds.

Poincaré-Bendixson Theory: Limit sets, local sections, Poincaré-Bendixson theorem and its applications (if time permits).

Suggested Texts:

1. Barreira. L. and Valls, C., Ordinary Differential Equations: Qualitative Theory, American Mathematical Society .
2. Birkhoff. G. and Rota. G.C., Ordinary Differential Equations, Wiley.
3. Coddington, E.A. and Levinson, N., Theory of Ordinary Differential Equations, McGraw-Hill.
4. Hirsch, M.W., Smale, S. and Devaney, R.L., Differential Equations, Dynamical Systems, and an Introduction to Chaos, Elsevier/Academic Press, Amsterdam.
5. Hsieh, P. and Sibuya, Y., Basic Theory of Ordinary Differential Equations, Springer-Verlag.
6. Perko, L., Differential Equations and Dynamical Systems, Springer-Verlag.
7. Simmons. G.F., Differentials Equations with Applications and Historical Notes, Tata McGraw-Hill.
8. Teschl. G., Ordinary Differential Equations and Dynamical Systems, American Mathematical Society.

3rd Semester

MA4101: Algebra III **Core course/4 credits**

This is course on fields and Galois theory.
Fields: Fields, field of fractions, field extensions, algebraic extensions, degree of an extension, splitting fields, normal extensions, separable extensions, finite fields.

Galois Theory: Galois extensions, automorphism groups and fixed fields, fundamental theorem of Galois theory and applications, cyclic extensions, cyclotomic polynomials, solvable groups, solvability by radicals, constructibility of regular polygons, transcendental extensions.

Suggested Texts:

1. Artin, M., Algebra, Prentice-Hall.
2. Artin, E., Algebra with Galois Theory, Courant Lecture Notes.

3. Gopalkrishnan, N.S., University Algebra, New Age International Press.

4. Lang, S., Algebra, Springer.

5. Morandi, P., Field and Galois Theory, Springer-Verlag.

MA4102: Functional Analysis **Core course/4 credits**

Normed Linear Spaces: Definitions, Banach spaces, Hilbert spaces, non-compactness of the unit ball in infinite dimensional normed linear spaces, quotient spaces.

Linear Maps: Boundedness and continuity, linear functionals, isometries.

Convexity: Hahn-Banach extension theorem, complex Hahn-Banach theorem, separation of convex sets, applications.

Completeness: Baire category theorem, Banach-Steinhaus theorem, open mapping theorem and closed graph theorem.

Duality: Dual spaces, Riesz representation theorem, reflexivity, weak topologies, weak convergence, weak compactness, Banach-Alaoglu theorem, adjoints and compact operators with examples, Volterra operators.

Hilbert Spaces: Bessel's inequality, complete systems, Gram-Schmidt orthogonalization, Parseval's identity, projections, orthogonal decomposition, bounded linear functionals in Hilbert spaces.

Spectral Theory: Spectrum, spectral theory of compact self-adjoint operators, Spectral theory of compact normal operators.

Suggested Texts:

1. Brezis, H., Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.

2. Bollabs, B., Linear Analysis: An Introductory Course, Cambridge University Press.

3. Conway, J.B., A Course in Functional Analysis (2nd Edition), Springer-Verlag.

4. Eidelman, Y., Milman, V. and Tsolomitis, A., Functional Analysis: An Introduction, American Mathematical Society.

5. Kesavan, S., Functional Analysis, Hindustan Book Agency.

6. Lax, P.D., Functional Analysis, Wiley-Interscience.

7. Limaye, B.V., Functional Analysis, New Age Publishers.

8. Rudin, W., Functional Analysis, McGraw-Hill.

9. Simmons, G.F., Topology and Modern Analysis, Tata McGraw-Hill.

MA4103: Analysis V **Core course/4 credits**

This is course on complex measures and Fourier series.
Prerequisite: Results proved in Functional Analysis (MA4102) course will be useful for the 'Fourier Series' part.

Signed Measures: Total variation measure, absolute continuity, Lebesgue decomposition, Radon-Nikodym theorem, Hahn decomposition theorem.

Convolution: Definition and basic properties, Young's inequality, mollifiers and approximation by smooth functions.

Differentiation Theory: Hardy-Littlewood maximal functions, Lebesgue differentiation theorem, Lebesgue points, absolutely continuous functions, fundamental theorem of calculus, Rademacher theorem.

Fourier Series: Fourier coefficients and series, summability, pointwise convergence of Fourier series, convergence of Fourier series in norm.

Suggested Texts:

1. Duoandikoetxea, J., Fourier Analysis, American Mathematical Society.
2. Evans, L.C. and Gariepy, R.F., Measure Theory and Fine Properties of Functions, CRC Press.
3. Folland, G.B., Real Analysis: Modern Techniques and Their Applications (2nd Edition), Wiley-Interscience.
4. Grafakos, L., Classical Fourier Analysis, Springer-Verlag.
5. Katznelson, Y., An Introduction to Harmonic Analysis, Cambridge University Press.
6. Rudin, W., Real and Complex Analysis, McGraw-Hill.

MA4104: Algebraic Topology **Core course/4 credits**

Fundamental Group: Review of quotient topology, path homotopy, definition of fundamental group, covering spaces, path and homotopy lifting, fundamental group of S^1, deformation retraction, Brouwer's fixed point theorem, Borsuk-Ulam theorem, van-Kampen's theorem, fundamental group of surfaces, universal covering space, correspondence between covering spaces and subgroups of fundamental group.

Homology Theory: Simplicial complexes and maps, homology groups, computation for surfaces.

Suggested Texts:

1. Hatcher, A., Algebraic Topology, Cambridge University Press.
2. Massey, W.S., A Basic Course in Algebraic Topology, Springer-Verlag.
3. Munkres, J.R., Elements of Algebraic Topology, Addison-Wesley.
4. Spanier, E.H., Algebraic Topology, Springer-Verlag.

MA5102: Partial Differential Equations **Core course/4 Credits**

First-order Equations: Method of characteristics and existence of local solutions.

Characteristic Manifolds and Cauchy Problem: Non-characteristic surfaces, Cauchy-Kowalevski theorem and uniqueness theorem of Holmgren.

Laplace Equation: Fundamental solution, harmonic function and its properties, Poisson's equation, Dirichlet problem and Green's function, existence of solution of the Dirichlet problem using Perron's method, introduction to variational method.

Heat Equation: Fundamental solution and initial-value problem, mean value formula, maximum principle, uniqueness and regularity, nonnegative solutions, Fourier transform methods.

Wave Equation: d'Alembert's formula, method of spherical means, Hadamard's method of descent, Dumahel's principle and Cauchy problem, initial-boundary-value problem, Fourier transform methods.

Suggested Texts:

1. Evans, L.C., Partial Differential Equations, American Mathematical Society.
2. Folland, G., Introduction to Partial Differential Equations, Princeton University Press.
3. Gilbarg, D. and Trudinger, N., Elliptic Partial Differential Equations of Second Order, Springer-Verlag.
4. Han, Q., A Basic Course in Partial Differential Equations, American Mathematical Society.
5. John, F., Partial Differential Equations, Springer-Verlag.

6. McOwen, R.C., Partial Differential Equations: Methods and Applications, Pearson Education.

7. Renardy, M. and Rogers, R., An Introduction to Partial Differential Equations, Springer-Verlag.

4th Semester

MA4203: Probability II **Core course/4 credits**

This is an introduction to modern probability theory and may be treated as a sequel to basic probability (similar to MA2202: Probability I in the BS-MS program).

Quick review of concepts and results (without proof) from basic discrete and continuous random variables, 1-1 correspondence between distribution functions and probabilities on $\mathbb{R}$, examples of probability measures in Euclidean space, a metric on the space of probability measures on $\mathbb{R}^d$, expectation and the convergence theorems, independence, Borel-Cantelli lemma, weak and strong laws in the *iid* cases, Kolmogorov's 0-1 law and three-series theorem, various modes of convergence, infinite products, Kolmogorov's consistency theorem, characteristic functions, uniqueness, inversion theorem, Levy continuity theorem, proof of CLT for the *iid* case with finite variance, martingales, infinitely divisible laws and stable laws.

Suggested Texts:

1. Athreya, K.B. and Lahiri, S.N., Measure Theory and Probability Theory, Springer.

2. Ash, R. and Dolans-Dade, C.A., Probability and Measure Theory, Academic Press

3. Billingsley, P., Probability and Measure, John Wiley.

4. Borkar, V.S., Probability Theory : An Advanced Course, Springer.

5. Chung, K.L., A Course in Probability Theory, Elsevier.

6. Durrett, R., Probability : Theory and Examples, Cambridge University Press.

7. Gut, A., Probability : A Graduate Course, Springer.

8. Loève, M., Probability Theory, Vols. I & II, Springer.

9. Parthasarathy, K.R., Introduction to Probability and Measure, Hindustan Book Agency.

MA4205: Differential Geometry **Core course/4 credits**

Basic Theory: Topological manifolds, examples, differentiable manifolds and maps, immersed and imbedded manifolds, submanifolds, partitions of unity, compact manifolds as closed submanifolds of $\mathbb{R}^n$.

Tangent Space and Vector Fields: Definition of tangent vector as equivalence class of curves and derivations, tangent spaces and their mappings, tangent bundle, vector fields, integral curves, complete vector fields, Lie derivative and connection with Lie bracket of vector fields.

Differential Forms and Integration: Wedge product, Exterior differentiation: definition, axiomatic treatment and coordinate invariance, closed and exact forms, Poincaré lemma, review of classical line and surface integrals, integration on manifolds, orientation, Stokes' theorem, integration by parts.

Suggested Texts:

1. Guillemin, V. and Pollack, A., Differential Topology, AMS Chelsea.
2. Hirsch, M.W., Differential Topology, Springer.
3. Kumaresan, S., A Course in Differential Geometry and Lie Groups, Hindustan Book Agency.
4. Lee, J.M., Introduction to Smooth Manifolds, Springer-Verlag.
5. Milnor, J,W., Topology from the Differentiable Viewpoint, Princeton University Press.
6. Mukherjee, A., Topics in Differential Topology, Hindustan Book Agency.
7. Spivak, M., A comprehensive Introduction to Differential Geometry, Vol. I, 3rd Edition, Publish or Perish.
8. Tu, L.W., An Introduction to Manifolds, Universitext, Springer-Verlag.

MAxxxx: Departmental Elective I **Elective course/4 credits**

MAxxxx: Departmental Elective II **Elective course/4 credits**

MA4210: MS Project **Core course/8 credits**

List of Electives

MA4204: Representation Theory of Finite Groups **Elective course/4 credits**

Representations of Finite Groups: Definitions, Schur's theorem, characters, group algebra, Maschke's theorem, simple modules over group algebras, inner products of characters, the number of irreducible characters, character tables and orthogonality relations, Burnside's two-prime theorem, Induced representation, Frobenius reciprocity, construction of character tables of $GL(2, k)$, $SL(2, k)$, $PGL(2, k)$ where k is a finite field.

Further Topics: Brauer's theorem on induced characters, representation of symmetric groups, Young diagrams and Frobenius' character formula.

Suggested Texts:

1. Fulton, W. and Harris, J., Representation Theory : A First Course, Springer-Verlag.
2. James, G. and Liebeck, M., Representations and Characters of Groups, Cambridge University Press.
3. Musili, C., Representations of Finite Groups, Hindustan Book Agency.
4. Serre, J.P., Linear Representation of Finite Groups, Springer.

MA4208: Independent Study I **Elective course/4 credits**

This course is currently available to 4th year DMS BS-MS students as well as 2nd year DMS MS students as an elective. The guidelines are as follows:

1. *Choosing a mentor (for DMS 2nd year MS students)*

   a. A student may choose to do an independent study course with a DMS faculty mentor from IISER Kolkata.

   b. The choice of the mentor should be conveyed to the course co-ordinator(s) (as displayed on welearn) within the **first seven days** of the commencement of the semester. This should be done via an e-mail to the course co-ordinator(s) with a copy to the mentor.

2. *Contact hours & responsibility*

   a. It is a **4 credit** course. The responsibility is on the students to meet the mentor for at least **2 contact hours** per week. The meeting mode and place may be mutually decided by the instructor and the student.

b. Failure to meet the mentor for at least **three weeks** may result in failure for the course.

3. *Syllabus & Grading*

a. As this is a course, a syllabus has to be decided by the mentor and notified to the course co-ordinator(s) (as displayed on welearn) within the **first seven days** of the commencement of the semester. Only after the approval of this syllabus by DMS UGAC, in consultation of course co-ordinator(s), should the student be allowed to register for this course.

b. There will be **two** written examinations: mid-sem and final. The entire grade for the course will be based on these two exams.

MA5202 : Algebraic Geometry **Elective course/4 credits**

The projective plane, Bezout's theorem, application (addition law on cubic curves) and proof, affine varieties, quasi-projective varieties, images of projective varieties, dimension of varieties, nonsingular projective curves, divisors, Picard group, hyper-elliptic curves, differentials, the canonical divisor, genus of a curve, Riemann-Roch theorem, Riemann-Hurwitz formula.

Suggested Texts:

1. Fulton, W., Algebraic Curves : An Introduction to Algebraic Geometry, Addison-Wesley.
2. Harris, J., Algebraic Geometry : A First Course, Springer-Verlag.
3. Kendig, K., Elementary Algebraic Geometry, Springer-Verlag.
4. Musili, C., Algebraic Geometry for Beginners, Hindustan Book Agency.
5. Shafarevich, I.R., Basic Algebraic Geometry, Springer.

MA5203: Topics in Operator Theory **Elective course/4 credits**

Topics to be chosen from the following texts:

1. Agler, J. and McCarthy, J.E., Pick Interpolation and Hilbert Function Spaces, American Mathematical Society.
2. Conway, J.B., Subnormal Operators, Mathematical Surveys and Monographs, American Mathematical Society.

3. Paulsen, V., Completely Bounded Maps and Operator Algebras, Cambridge University Press.

4. Sz-Nagy, B., Foias, C., Bercovici, H. and Kerchy, L., Harmonic Analysis of Operators on Hilbert Space, Springer.

MA5204 : Several Complex Variables **Elective course/4 credits**

Holomorphic functions, separately holomorphic functions, Hartog's theorem, homomorphically convex domains and its characterisations, inhomogeneous Cauchy-Riemann equation, Hartog's phenomenon, domains of holomorphy, plurisubharmonic functions, pseudoconvex domains and its characterisations, Levi problem, solution of $\bar{\partial}$-problem by Hörmander's method.

Suggested Texts:

1. Hormander, L., An Introduction to Complex Analysis in Several Variables, Elsevier.

2. Krantz, S., Function Theory of Several Complex Variables, AMS Chelsea Publishing.

3. Ohsawa, T., Analysis of Several Complex variables, Mathematical Monographs, American Mathematical Society.

MA5205: Advanced Partial Differential Equations **Elective course/4 credits**

Sobolev Spaces: Distribution theory, Sobolev spaces, embedding theorems, Rellich's lemma, trace theorem.

Elliptic Equations: Second order elliptic equations, weak formulation, Lax-Milgram lemma, existence.

Hamilton-Jacobi Equations: Hopf-Lax formula, weak solution of Hamilton-Jacobi equation and its uniqueness.

Conservation Laws: Weak solutions, Rankine-Hugoniot condition, shocks, Lax-Oleinik formula, entropy condition and uniqueness of entropy solution.

Suggested Texts:

1. Attouch. H., Buttazzo. G. and Michaille. G., Variational Analysis in Sobolev and BV Spaces: Applications to PDEs and Optimization, SIAM.

2. Brezis. H., Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.

3. Evans. C.L., Partial Differential Equations, American Mathematical Society.
4. Kesavan. S., Topics in Functional Analysis and Applications, Wiley.
5. Leoni. G., A First Course in Sobolev Spaces, American Mathematical Society.

MA5206: Topics in Analysis **Elective course/4 credits**

Relevant papers will be given by the Instructor.

MA5207: Topology and Geometry **Elective course/4 credits**

Algebraic Topology: Poincaré Duality, Kunneth product formula, universal coefficient theorem.

Differential Topology: Transversality and Morse-Sard theorem, distributions and integrability, Frobenius theorem.

Euler Class: Orientation, intersection number, Euler characteristic, Lefschetz fixed point theorem, index of vector field, Poincaré-Hopf index theorem, Euler form and Euler class, Euler class of oriented manifolds via Poincaré duality.

Suggested Texts:

1. Hatcher, A.W., Algebraic Topology, Cambridge University Press.
2. Milnor, J.W. and Stasheff, J., Characteristic Classes, HBA
3. Guillemin, V. and Pollack, A., Differential Topology, Prentice-Hall
4. Hirsch. M., Differential Topology, Springer
5. Bott, R. and Tu, L.W., Differential Forms and Algebraic Topology, Springer

MA5213: Sobolev Spaces: Theory and Applications
Elective course/4 credits

Sobolev Spaces: Weak derivative, Sobolev spaces, approximation by smooth functions, approximations up to the boundary, extension of Sobolev functions, embedding theorems, compact embeddings, Rellich Theorem, Poincare inequalities, traces of Sobolev functions.

Elliptic PDE (L^2 theory): Elliptic equations, Lax-Milgram Theorem, existence of weak solutions, regularity of solutions, maximum principles, eigenvalue problem.

Suggested Texts:

1. Adams, R. A. and Fournier, J. J. F., Sobolev Spaces. Second edition. Pure and Applied Mathematics (Amsterdam), 140, Elsevier/Academic Press, Amsterdam, 2003.

2. Evans, L. C., Partial Differential Equations. Second Edition, AMS, 2010.

3. Evans, L. C. and Gariepy, R. F., Measure Theory and Fine Properties of Functions. CRC Press, 1992.

4. Gilbarg, D. and Trudinger, N., Elliptic Partial Differential Equations of Second Order. Second Edition, Springer, 1983.

5. Kesavan, S., Topics in Functional Analysis and Applications. Wiley Eastern Limited, 1989.

6. Leoni, G., A First Course in Sobolev Spaces. AMS, 2009

MA5217: Commutative Algebra **Elective course/4 credits**

Rings and Ideals: (Revision of Ideals, Quotient rings, Prime and Maximal ideals) Nil radical and Jacobson radical, operations on ideals, extension and contraction. Chinese remainder Theorem, Prime avoidance Lemma.

Modules: Revisions of modules, Exact sequences, Tensor product of modules, Exactness properties of the Hom and Tor, Flat and faithfully flat modules.

Localizations: Localization of rings/modules, Universal property of localization, Extended and contracted ideals and prime ideals under localization, Localization and quotient, Exactness property, Local properties. 4. Integral Extensions. Integral dependence, The Going up theorem, Integrally closed domains, The Going down theorem.

Primary Decomposition and Associated Primes.

Chain Conditions and Noetherian Rings.

Suggested Texts:

1. M. F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra, Addision Wesley (1969).

2. N. S. Gopalakrishnan, Commutative Algebra,Oxonian Press (1984).

3. M. Reid, Undergraduate Commutative Algebra, LMS Student Texts (29), Cambridge Univ. Press (1995).

4. Matsumura, H., Commutative Ring Theory, Cambridge Studies in Advanced Mathematics, 1989.

5. Eisenbud, D., Commutative Algebra with a view towards Algebraic Geometry, GTM 151, Springer-Verlag, 1995.